

Extending the unambiguous range of two-color interferometers

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The unambiguous distance measurement range in two-color interferometry is generally understood to be limited to the equivalent or synthetic wavelength, which is inversely proportional to the wavelength separation of the two colors. Here it is shown that one may extend the unambiguous range well beyond this limit by using optical phase information to determine the synthetic-wavelength fringe order.

1. Introduction

A well-known problem in interferometric metrology involves the measurement of distances larger than the source wavelength. The essential difficulty relates to the interferometric fringe order, which cannot be determined unambiguously from a single measurement of phase. Because one of the earliest uses of interferometry was the comparison and calibration of length standards, the rapid determination of fringe order was one of the first problems to be resolved. The method employed by Benoit in 1898 to measure 1-cm étalons involved a Michelson interferometer and the four principal visible colors of cadmium emissions.^{1,2} The Michelson-Benoit method of excess fractions consists of a comparison of the fractional fringes for each of the colors with a table of values calculated on the basis of the known wavelengths of the red, green, blue, and violet emissions of cadmium. Essentially the same method found its way into standards laboratories in the 1930's and was applied to the routine measurement of gauge blocks to an accuracy of 25 nm through the use of a purely visual interpretation of interference patterns.^{3,4}

Recently, the resurgence of multiple-color techniques in distance measurement and surface profiling has introduced new methods of calculating distances that do not rely on tables of excess fringes or special slide rules that can only be used with a particular set of wavelengths. Perhaps the simplest of these tech-

niques is based on the concept of an equivalent or synthetic wavelength, which corresponds to the spatial beat frequency in an interferogram obtained with two colors. A pleasing example of synthetic wavelengths can be found in two-wavelength holography,^{5,6} in which the synthetic wavelength corresponds to the contour intervals of constructive interference in a reconstructed holographic image.

The use of synthetic wavelengths has been widely accepted in many different forms of multiple-color interferometry, both as a general computational alternative to the method of excess fractions⁷ and as an approach to the resolution of phase ambiguities in three-dimensional phase-modulation interferometric imaging.^{8,9} The technique is easy to use and understand, and as a consequence almost every modern two-color interferometer employs an analysis based on synthetic wavelengths. This includes instruments for distance measurement,^{10,11} optical testing,^{12,13} microscopy,¹⁴ inspection of integrated circuits,¹⁵ manufacturing,¹⁶ and fiber sensing.¹⁷ The only limitation of the synthetic-wavelength method, apart from practical difficulties of implementation, is that the unambiguous range of measurement is limited to the synthetic wavelength. Outside of this range, the synthetic fringe order is indeterminate and the measurement is useless. However, as shown here, it is almost always possible for one to extend greatly the unambiguous range of two-color interferometers beyond the limits of the synthetic wavelength, simply by changing the way the phase data is analyzed.

2. Two-Color Interferometry

Here the basic principles of two-color interferometry are covered. The nature of the problem is clarified and the terminology and notation that are used in Section 3 are introduced.

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A typical interferometer such as the Michelson amplitude-division interferometer shown in Fig. 1 provides a sensitive measure of variations in optical path difference. Calculation of a round-trip optical path difference, L , is based on the measured phase $-\pi < \phi < \pi$ and the optical wavelength λ :

$$L = \left(n + \frac{\phi}{2\pi} \right) \lambda. \quad (1)$$

Because the phase is determined from an inverse trigonometric function, the same measured value of ϕ will be repeated at path-length intervals equal to

$$R_n = n\lambda, \quad (2)$$

where n is an integer that cannot be determined from the phase measurement alone. If one is interested only in relative displacements, or if one is using interferometry to profile a smooth, continuous surface, then it is often possible to count fringes and in this way achieve submicrometer accuracy over ranges of a meter or more.¹⁸⁻²⁰ However, in the absence of some other information regarding the optical path difference, the interferometer can only make individual measurements unambiguously over a range limited to the optical wavelength.

In two-color interferometry a second wavelength, λ_2 , with an associated phase ϕ_2 and integer fringe order n_2 is introduced. Using this notation, one sees that the Michelson-Benoit method of excess fractions consists of determining mutually consistent values for the integer fringe orders, n and n_2 , given the measured fractional fringe orders, $\phi/2\pi$ and $\phi_2/2\pi$. One can reduce the computation procedure for this method to calculating a large number of possible distances for the given fractional fringe orders and then observing which values are in closest agreement. This method can rapidly become tedious if a wide variety of distances have to be measured, or if the wavelengths are not always exactly the same.

It has become common practice for one to make the data processing in two-color interferometry more rapid and intuitive by defining a synthetic wave-

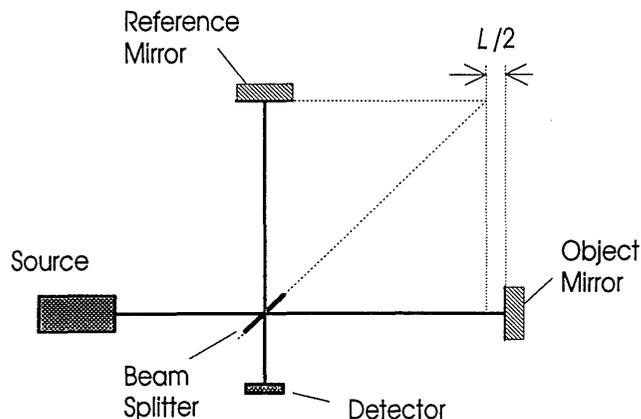


Fig. 1. Michelson interferometer used for measuring the optical path difference between two mirrors.

length Λ , defined by the spatial beat period for a two-color interference pattern. The corresponding synthetic phase is

$$\Phi = \phi - \phi_2, \quad (3)$$

constrained by $-\pi < \Phi \leq \pi$. Using this concept and assuming a perfectly compensated interferometer, one can obtain an optical path difference estimate, L' , from

$$L' = \left(N + \frac{\Phi}{2\pi} \right) \Lambda, \quad (4)$$

where N is the integer synthetic fringe order and the synthetic wavelength is given by

$$\Lambda = \frac{\lambda_2 - \lambda}{\lambda\lambda_2}. \quad (5)$$

If $N = 0$, then one can make an estimate n' of the optical wavelength fringe order by substituting Eq. (4) into Eq. (1) and rearranging:

$$n' = \frac{1}{2\pi} \left(\frac{\Phi\Lambda}{\lambda} - \phi \right). \quad (6)$$

The final optical path difference measurement is then

$$L = \left[\text{int}(n') + \frac{\phi}{2\pi} \right] \lambda, \quad (7)$$

where the function $\text{int}(\)$ returns the nearest integer to its argument. The unambiguous measurement range has now been extended to the synthetic wavelength Λ , which may be much larger than λ .

The limitation of the synthetic-wavelength method is that the same synthetic phase, Φ , will repeat itself at path differences

$$R_N = N\Lambda. \quad (8)$$

Thus the unambiguous range interval for this method is defined by $|L| < \Lambda/2$. It is generally accepted that the only way to extend this range is either to increase the synthetic wavelength Λ or to incorporate additional optical, electrical, or mechanical means of removing the synthetic-wavelength phase ambiguity. This limitation restricts the choice of source and detection methods available for implementation of two-color interferometry, and it applies to all interferometers employing two colors for the purposes of measuring distances unambiguously over ranges larger than an optical wavelength.

At this point one should note that, in the original method of excess fractions as described by Benoit, there is no explicit restriction on the achievable unambiguous measurement range. The only requirement is that the collection of phases corresponding to the different colors employed be unique within the ability of the operator to measure excess fractions. For example, Table 1 shows the excess fractions that

Table 1. Excess Fractions $\phi/2\pi$ and $\phi_2/2\pi$ Corresponding to the Green and Red Wavelengths of Cadmium

L (μm)	$\phi/2\pi$	$\phi_2/2\pi$	$\Phi/2\pi$
0.00	0.00	0.00	0.00
1.21	0.38	-0.12	0.50
2.42	-0.24	-0.24	0.00
3.63	0.14	-0.36	0.50
4.84	-0.48	-0.48	0.00
6.05	-0.10	0.40	-0.50
7.28	0.28	0.28	0.00
8.47	-0.34	0.16	-0.50

one would observe by using the green (508.5824-nm) and red (643.8472-nm) cadmium emissions over a series of distances near zero. A careful observer, able to measure an excess fraction with an accuracy of 0.05 (1/20 of a fringe), would easily be able to distinguish between these distances, in spite of the fact that the synthetic wavelength is only 2.42 μm . A more extensive table of values confirms that the classical method of excess fractions for these two colors has an unambiguous range of approximately four times the synthetic wavelength, or nearly 10 μm . Evidently, something is lost when one translates to the synthetic-wavelength picture of two-color interferometry.

3. Increasing the Unambiguous Range

The purpose of the following discussion is to find a way to recover the total achievable unambiguous range in a two-color interferometer while retaining the basic concept of a synthetic wavelength. Within this conceptual framework, our objective will be to remove the ambiguity in the synthetic fringe order by using the optical phase information that is discarded in conventional approaches to reducing two-color fringe information. Knowledge of the synthetic fringe order over an extended unambiguous range will then permit a final calculation of distance that is more flexible than the classical method of excess fractions while full use is made of all the available information.

First let us recall that the optical phase ϕ repeats at intervals $R_n = n\lambda$, whereas the synthetic phase Φ repeats at intervals $R_N = N\Lambda$. In conventional synthetic-wavelength algorithms there is a natural limit of $\pm\Lambda/2$ imposed on the achievable unambiguous range, based on the fact that synthetic phase Φ repeats at intervals equal to synthetic wavelength Λ . However, the phase couple (Φ, ϕ) , composed of optical phase ϕ and synthetic phase Φ , repeats at intervals for which $R_N = R_n$. These intervals may be much larger than synthetic wavelength Λ .

When the two repeat distances, R_N and R_n , are set equal to each other, we find that

$$\Lambda = \frac{n_R}{N_R} \lambda, \quad (9)$$

where n_R and N_R are the values of integers n and N for which the couple (Φ, ϕ) repeats itself. Normally

coefficient n_R/N_R will be some noninteger rational number, and integers n_R and N_R may have to be large to approximate it. If $\Lambda = 10.1 \times \lambda$, for example, then $n_R = 101$ and $N_R = 10$. The repeat distance $N_R\Lambda$ for this example is ten times the synthetic wavelength.

The idea then is for us to extend the unambiguous range by making full use of the spatial evolution of the phase couple (Φ, ϕ) . In deriving this method, let us first recall that Eq. (6) provides an estimate n' of the optical wavelength integer fringe order that is valid over a range of $\pm\Lambda/2$. Both the synthetic and optical phases Φ and ϕ appear in this equation. Now if L is larger than Λ , fractional errors will be introduced into Eq. (6) that are equal to N times the noninteger part of the ratio of Λ to λ . These fractional errors are therefore an indication of the integer synthetic fringe order. From these observations we may derive a formula that yields an estimate N' that we can use to extend the unambiguous range:

$$N' = -\frac{n' - \text{int}(n')}{\Lambda/\lambda - \text{int}(\Lambda/\lambda)}. \quad (10)$$

This result leads to a corrected estimate n'' of the optical fringe order valid over the extended range:

$$n'' = \frac{1}{2\pi} \left(\frac{\Phi\Lambda}{\lambda} - \phi \right) + \frac{\Lambda}{\lambda} \text{int}(N'). \quad (11)$$

We can use this value in a final calculation of the distance by first obtaining

$$L'' = \left[\text{int}(n'') + \frac{\phi}{2\pi} \right] \lambda, \quad (12)$$

and then constraining the final answer to the extended range $\pm N_R\Lambda/2$ by using

$$L = L'' - N_R\Lambda \text{int} \left(\frac{L''}{N_R\Lambda} \right). \quad (13)$$

We have extended the unambiguous range by a factor of N_R by simply changing the way the data are analyzed. The extended-range algorithm is summarized in Appendix A.

The range multiplier N_R for a particular combination of wavelengths can be calculated from the following formula:

$$N_R = \left\lceil \text{int} \left[\frac{1}{\Lambda/\lambda - \text{int}(\Lambda/\lambda)} \right] \right\rceil. \quad (14)$$

For example, if we select the green and red cadmium emissions, the synthetic wavelength is 2.42 μm and the red wavelength is 0.644 nm. The extended-range multiplier calculated with Eq. (14) is equal to four and the extended range is 10 μm , exactly as we expect from inspection of Table 1.

Another example of how the unambiguous range can be extended is illustrated in Fig. 2. An optical wavelength of $\lambda = 612.0$ nm and a second wavelength of $\lambda_2 = 648.0$ nm were chosen, thus providing a

synthetic wavelength of $\Lambda = 11.147 \mu\text{m}$. In a computer simulation, various modulo 2π phases, ϕ and ϕ_2 , were generated for a range of round-trip optical path differences L . Proceeding from Eqs. (6), (10), (11), and (12), we can recover the original distances without ambiguity over a range of $5\Lambda = 56 \mu\text{m}$. For comparison, the results of the conventional synthetic-wavelength algorithm involving only Eqs. (6) and (7) are also presented in Fig. 2. The conventional algorithm can only be used over a range of $11 \mu\text{m}$.

There is, of course, a catch. Actually there are several catches. An obvious problem is the increase in data-processing time, which is a serious consideration in three-dimensional interferometric imaging. In addition, two special requirements have to be met in order for the extended-range algorithm to work. First, the synthetic wavelength must satisfy Eq. (9), which may involve the special selection or adjustment of one or both optical wavelengths λ or λ_2 so that we achieve the particular range multiplier N_R desired. Second, the signal-to-noise ratio, the phase-measurement precision, and the wavelength-stability requirements for obtaining an extended range are all proportionally more stringent for increasing values of the range multiplier N_R (see Appendix B). Another consideration is that the degree of achromatization of the optical components in the interferometer may not accommodate a large spectral separation of the colors. In that case, it may be more desirable for us to increase the unambiguous range by making the synthetic wavelength longer, which we achieve by bringing the two source wavelengths closer together.

This brings up the question of whether there is any practical advantage to using the extended-range method. The answer is yes, if there is some obstacle involving the selection of source wavelengths that

makes it undesirable for one to increase the unambiguous measurement range by making the synthetic wavelength longer. For example, two optical wavelengths might be generated by two different kinds of inexpensive lasers, light-emitting diodes, or interference filters, which might become more expensive or difficult for one to obtain if a significantly different wavelength separation is required to increase the synthetic wavelength. In addition, some interferometers use prisms or gratings to multiplex two colors,^{21,22} which are easier for one to separate spectrally if the synthetic wavelength is small.

For a practical example, let us consider the hypothetical case of a two-color interferometer illuminated by a 632.8-nm He-Ne gas laser combined with a tunable laser diode operating at 780 nm. Such a combination may be useful in the modification of existing interferometers based on He-Ne lasers, for the purpose of measuring discontinuous step heights in a target surface or the metrology of rough surfaces, or in the calculation of the radius of curvature of an optical component under test. We easily separate the two colors before detection by using interference filters or a grating. However, the resulting synthetic wavelength of $3.35 \mu\text{m}$ is too small for many applications if the unambiguous range is limited to this value. Because there is a much wider choice of laser diodes in the near infrared, including wavelength-stabilized and high-power devices, it would be undesirable for us to lower substantially the diode wavelength simply to extend the unambiguous range. A more attractive alternative under these circumstances may be for us to use an extended-range algorithm without changing the sources. Because the laser diode is tunable over a small range, we could select the desired range multiplier N_R for the particular application, using the procedure outlined in Appendix B.

4. Conclusion

The objective of this paper was to derive an analytical approach to data processing for two-color interferometry that is valid over a larger measurement range than the conventional synthetic-wavelength techniques currently in use. The ability to measure beyond the synthetic wavelength increases the functionality of interferometric measuring tools, making it possible for one to use sources whose composite colors are widely spaced in wavelength, even in applications requiring comparatively large unambiguous range. The approach presented here involves the extension of the unambiguous range of two-color interferometry to a distance $N_R \Lambda$ by consideration of the couple (Φ, ϕ) , so that one retains the intuitive concepts of a synthetic wavelength Λ and a range multiplier N_R . Of course, it is possible for one to construct equivalent methods that make full use of the available phase information without relying on the terminology or algebraic manipulations in this paper. The essential conclusion is that there is

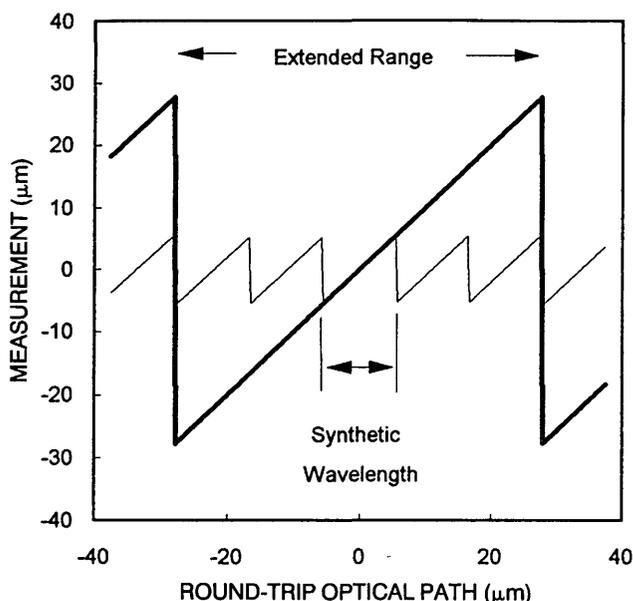


Fig. 2. Results of a computer simulation in which the extended unambiguous range and the restricted range of the conventional synthetic-wavelength methods are compared.

almost always a way in principle to distinguish one synthetic fringe from another, and the unambiguous range in a two-color interferometer need not be limited to the synthetic wavelength.

Appendix A: Extended-Range Algorithm

The following computational procedure is based on independent measurements of optical phases ϕ and ϕ_2 at wavelengths λ and λ_2 , and it is valid over a range of $\pm N_R \Lambda / 2$. Synthetic wavelength Λ and range multiplier N_R are characteristics of the source related to the wavelength separation of the two colors in the interferometer.

Step 1. Measure optical phases ϕ and ϕ_2 , and calculate difference Φ .

Step 2. Use Eq. (6) to obtain an initial estimate n' of the optical fringe order. (The fractional error in this estimate relates to the synthetic fringe order.)

Step 3. Use Eq. (10) and n' to estimate the synthetic fringe order N' .

Step 4. Use Eq. (11) to obtain a corrected estimate n'' of the optical fringe order.

Step 5. Use Eq. (12) to calculate an intermediate value L'' .

Step 6. Use Eq. (13) to constrain the final answer L to the $\pm N_R \Lambda / 2$ range.

Appendix B: Error Analysis

The purpose of the extended two-color algorithm is to determine the fringe order in an interferometer and in this way extend the operational range of the instrument. Thus the final accuracy of the interferometer is the same as a single-color measurement, and we use whatever means are at our disposal for determining the phase. For example, from Eq. (1) we see that if the fringe order is correctly determined, then measurement uncertainty ΔL is

$$\Delta L / L = \Delta m (\lambda / L) + \Delta \lambda / \lambda, \quad (\text{B1})$$

where $\Delta m = \Delta \phi / 2\pi$ is the phase-measurement accuracy and $\Delta \lambda / \lambda$ is the wavelength stability. This looks rather promising and even a bit suspicious. The relative precision $\Delta L / L$ actually improves with distance. Clearly there must be a limit to how large a range we can achieve with this or any other two-color algorithm, and this limit is in fact determined by our ability to identify the fringe order correctly.

The extended-range algorithm provides an estimate n'' of the fringe order in Eq. (11), which for error analysis can be rewritten as

$$n'' = \frac{\Lambda}{\lambda} [M + \text{int}(N')] - m. \quad (\text{B2})$$

Here $M = \Phi / 2\pi$ and $m = \phi / 2\pi$. We calculate error $\delta n''$ in the fringe order by implicit differentiation of Eq. (14) and the definitions of synthetic phase Φ and

wavelength Λ found in Eqs. (3) and (5). Using

$$\delta \Lambda = \Lambda^2 \left(\frac{\delta \lambda}{\lambda^2} - \frac{\delta \lambda_2}{\lambda_2^2} \right), \quad (\text{B3})$$

$$L = [M + \text{int}(N')] \Lambda, \quad (\text{B4})$$

we see that the result is

$$\delta n'' = \left(L \frac{\delta \lambda}{\lambda^2} + \delta m \right) \left(\frac{\Lambda}{\lambda} - 1 \right) - \left(L \frac{\delta \lambda_2}{\lambda_2^2} + \delta m_2 \right) \left(\frac{\Lambda}{\lambda} \right). \quad (\text{B5})$$

Assuming totally uncorrelated errors and letting $\Delta \lambda_2 = \Delta \lambda$ and $\Delta m_2 = \Delta m$, we see that the final result in terms of measurement uncertainty is

$$\Delta n'' = \left(\frac{2\Lambda}{\lambda} - 1 \right) \left(\frac{\Delta \lambda}{\lambda} \frac{L}{\lambda} + \Delta m \right). \quad (\text{B6})$$

This uncertainty must be less than half of a fringe order in magnitude, i.e.,

$$\Delta n'' < 0.5. \quad (\text{B7})$$

This result shows that it is progressively more difficult for us to identify the fringe order successfully in a two-color interferometer as the optical path difference L is increased. This is true for both the extended-range and conventional synthetic-wavelength algorithms.²³ Thus phase-measurement precision Δm and wavelength-stability requirement $\Delta \lambda$ for obtaining an extended range are both proportionally more stringent for increasing values of range multiplier N_R .

An additional requirement of the extended-range algorithm is that the synthetic-wavelength fringe order must be correctly identified through the use of Eq. (10). As it turns out, this is an even more difficult condition to satisfy than that in Eq. (B6). The synthetic fringe order estimate in Eq. (10) is

$$N' = - \frac{n' - \text{int}(n')}{\Lambda / \lambda - \text{int}(\Lambda / \lambda)}. \quad (\text{B8})$$

Equation (B8) requires an estimate n' of the optical fringe order, which from Eq. (6) and appropriate changes in notation looks like

$$n' = \frac{M \Lambda}{\lambda} - m. \quad (\text{B9})$$

Further, for the purposes of error analysis, it is worthwhile for us to note that the denominator in Eq. (B8) is of the order of $1/N_R$. After some labor we find that

$$\Delta N_R \approx N_R \Delta n''. \quad (\text{B10})$$

This uncertainty must be less than 0.5 for us to identify the synthetic fringe order correctly. Consequently, from the point of view of error analysis,

there is no advantage to the extended-range algorithm over traditional methods. As noted in the main text here, the extended-range method becomes attractive when there is some obstacle involving the selection of source wavelengths that makes it undesirable for one to increase the unambiguous measurement range by making the synthetic wavelength longer.

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