Stroboscopic white-light interference microscopy

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The principle of stroboscopic motion freezing of oscillating objects extends directly to interference microscopes that use coherence as part of the measurement principle. Analysis shows, however, that the fringe contrast loss for out-of-plane motion in stroboscopic interferometry is a wavelength-dependent phenomenon, which can alter the apparent nominal center wavelength of the white-light source. As in monochromatic systems, the key adjustable parameter is the duty cycle, equal to the product of the vibrational frequency and the pulse width. This theoretical study provides detailed graphs of expected errors as a function of the duty cycle, including fringe contrast loss, apparent wavelength shift, and measurement error.

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1. Introduction: Stroboscopic Interferometry

Light pulsed synchronously with single-tone vibration causes an object to appear to stand still. This stroboscopic principle has been applied to holography and to interferometers[1,2] and was demonstrated for interference phase-shifting microscopy of microstructures by Eguchi and Okuma[3] and Nakano *et al.*[4] in the early 1990s. Recently adapted to modern commercial microscopes, e.g., by Gutierrez *et al.*,[5] Novak *et al.*,[6] and Bosseboeuf and Petigrand,[7] it is often marketed today as a dynamic measurement option for microelectromechanical systems (MEMS) and has found many applications for microsystems analysis.[8]

For stroboscopic interferometry, one identifies two basic categories of object vibration: in-plane and out-of-plane motions. Out-of-plane motions, in which the object moves in a direction that modulates the optical path difference of the interferometry, require stroboscopy to prevent blurring of the interference fringes and to allow profiling at specific phases of the vibrational motion. In-plane motions, in which the object moves laterally with respect to the optical axis, require stroboscopy to prevent blurring of the image.

In the following sections, I will concentrate on characterizing the out-of-plane motions, i.e., the up and down vibrations in a conventional vertical microscope configuration. It is generally understood that there are limitations to stroboscopic interferometry of out-of-plane motions related to the time width of each pulse. If the pulse is too long, for example, the object moves too far and the interference fringes will be blurred. There is also the potential for systematic measurement errors that may distort the final object profile.

Error analyses for stroboscopic interferometry have appeared in the literature for monochromatic (e.g., laser-based) systems.[4,7] I extend the analysis here to white-light interferometry, which is an important application of the technique. The following theoretical study shows that in addition to the expected blurring effects, there are phenomena unique to the use of white light, such as the distortion of the apparent source spectrum. Such effects can be important in properly interpreting the results of the stroboscopic measurements of dynamical motions using the scanning white-light interferometry.

2. Analysis

The integrating effect of a stroboscope pulse alters the apparent profile of an object as well as the contrast of the interference fringes. The fringe contrast loss, in particular, is a wavelength-dependent effect. Therefore for white-light illumination, which comprises a range of wavelengths, it makes sense to examine what happens to each of the constituent wavelengths and then to reconstruct the signal. This approach is familiar in white-light interferometry.[9,10]

Starting in the usual way by incoherent superposition, we write that a signal I in an interferometer is the sum of individual interference signals g that oscillate with the piezoelectric transducer (PZT) scan

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position ζ at a spatial frequency K:

$$I(\zeta) = \int_0^\infty q'(K)\cos[(\zeta - h)K + \omega(K)]\mathrm{d}K, \quad (1)$$

where $q'(K)$ are weights, ω is an offset constant with respect to ζ, and h is the surface height reference to the $\zeta = 0$ zero-scan position. It will be convenient to rewrite the integral as an inverse Fourier transform, with the phase offset $\omega - hK$ incorporated in a complex weighting factor:

$$I(\zeta) = \int_{-\infty}^\infty q(K)\exp(i\zeta K)\mathrm{d}K, \quad (2)$$

where, for $K \geq 0$,

$$q(K) = \frac{q'(K)}{2}\exp[i\omega(K) - ihK], \quad (3)$$

and for $K < 0$,

$$q(K) = \frac{q'(-K)}{2}\exp[-i\omega(K) - ihK]. \quad (4)$$

Now we add the vibratory motion about the mean position h of the object at a frequency f:

$$\xi = \Lambda \sin(2\pi ft). \quad (5)$$

This usually rapid periodic motion ξ is added to the scan position ζ, so that Eq. (2) becomes

$$I(\zeta,\ \xi) = \int_{-\infty}^\infty q(K)\exp[i(\zeta + \xi)K]\mathrm{d}K. \quad (6)$$

Next we consider the light pulsing of the stroboscopic source. This gets complicated quickly if we do not make some simplifying assumptions. The first of these simplifications is that the pulse length is short enough so that we may neglect the PZT scan motion during each pulse. The next simplifying assumption is that the pulse frequency is high, so that there are a great number of pulses per camera frame, and picking up or losing a single pulse in any given frame is not a significant error source. This latter assumption is safe if the nominal number of pulses per frame is greater than the number of gray levels of the camera, so that a dropped pulse is equivalent to 1 bit of intensity noise. This sets the minimum vibrational frequency for an 8-bit camera to 256 times the frame rate. Finally, we assume that the pulse is perfectly rectangular in shape, so that signal integration during the pulse is a simple averaging phenomenon.

With these assumptions, we can integrate the vibrational motion over a time interval $\delta t = t_2 - t_1$ by first writing

$$I(\zeta) = \frac{1}{\delta t}\int_{t_1}^{t_2}\int_{-\infty}^\infty q(K)\exp(i\zeta K)\exp(i\xi K)\mathrm{d}K\mathrm{d}t, \quad (7)$$

and then assume that the scan position ζ is independent of time over the short interval δt:

$$I(\zeta) = \int_{-\infty}^\infty \left[\frac{1}{\delta t}\int_{t_1}^{t_2}\exp(i\xi K)\mathrm{d}t\right] q(K)\exp(i\zeta K)\mathrm{d}K. \quad (8)$$

This is a significant and important simplification. Equation (8) says that the effects of integrating the strobe pulse are separable from the phase offsets attributable to scan ζ, height h, and phase offset ω, meaning that there are no phase-dependent or cyclic profiling errors, as there are often with other types of imperfection that lead to fringe print-through. Rewriting Eq. (8) to emphasize this point,

$$I(\zeta) = \int_{-\infty}^\infty u(K)q(K)\exp(i\zeta K)\mathrm{d}K, \quad (9)$$

where

$$u(K) = \frac{1}{\delta t}\int_{t_1}^{t_2}\exp[i\Lambda K\sin(2\pi ft)]\mathrm{d}t. \quad (10)$$

This can be further recast into a more convenient form:

$$u(K) = \frac{1}{2\pi D}\int_{-\pi D}^{\pi D}\exp[i\Lambda K\sin(\varphi + \varphi_0)]\mathrm{d}\varphi, \quad (11)$$

where the duty cycle D is the pulse width δt divided by the pulse period $1/f$:

$$D = f\delta t. \quad (12)$$

We can now study the effects of the stroboscopic illumination of a vibrating object by evaluating the frequency-domain function Eq. (11).

3. Examples

The function $u(K)$ has interesting properties. From Fig. 1(a), it is apparent that the fringe contrast is most strongly degraded at those φ_0 for which the vibrating object passes through the $\xi = 0$ position at high velocity, e.g., $\varphi_0 = 0$. From Fig. 1(b), we see the complementary situation for which the fringe contrast is high but the measurement error is maximum, this occurring at those φ_0 corresponding to the largest deflections in the vibrational motion, e.g., $\varphi_0 = \pi/2$.

An important phenomenon in stroboscopic white-light interferometry is the wavelength dependence of the fringe contrast loss. Figure 2(a) quantifies this

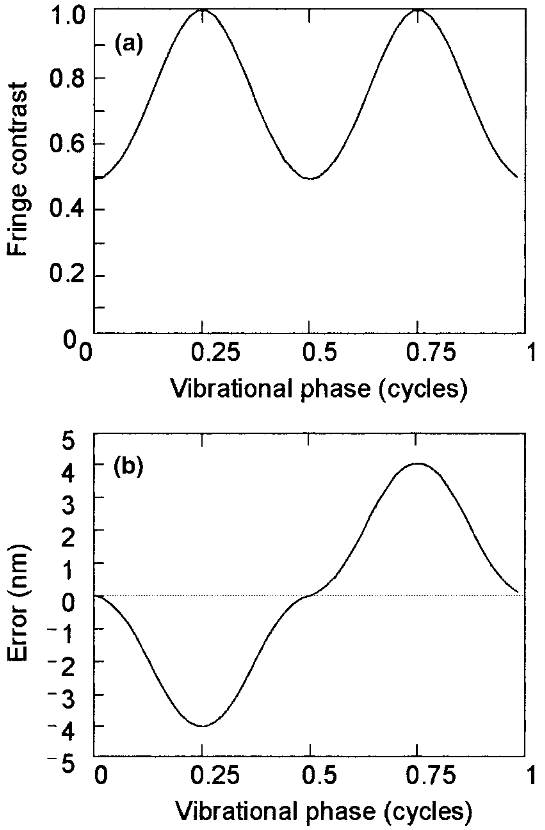

Fig. 1. (a) Variation in fringe contrast with vibrational phase φ. (b) Variation in measurement error with vibrational phase φ. Duty cycle of $D = 10\%$, vibrational amplitude of $\Lambda = 250$ nm.

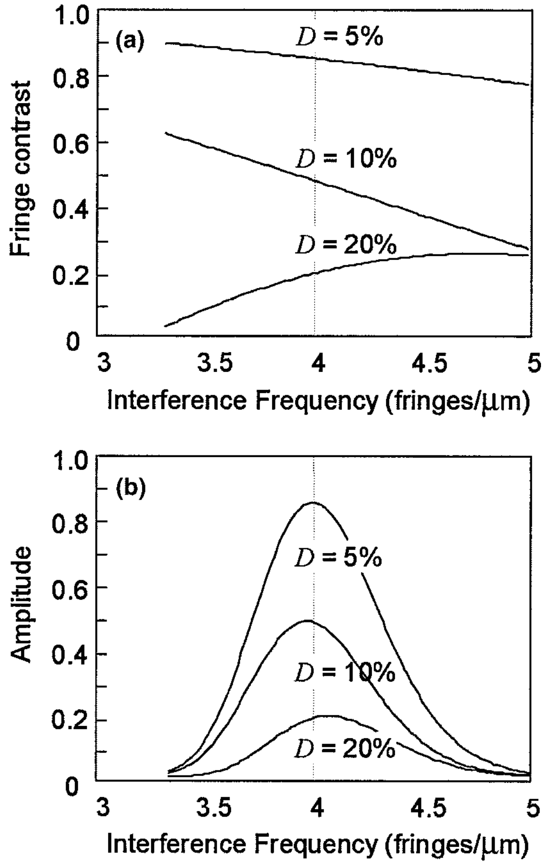

Fig. 2. (a) Dependence of the fringe contrast on interference spatial frequency. (b) Resultant distortion in the frequency domain of an interferometer signal, showing, in particular, a shift in the peak wavelength. Vibrational amplitude of 250 nm, vibrational phase of 0, and three different pulse duty cycles.

phenomenon. Figure 2(b) shows that this effect alters the perceived nominal wavelength of the light source. This can be an issue for any technique that makes a fixed, *a priori* assumption about the mean wavelength λ_0. Note that the wavelength shift would depend as well on the vibrational phase φ.

At the vibrational phase $\varphi_0 = \pi/2$, we observe in Fig. 3(a) a spatial-frequency dependence on the phase error that is very nearly linear. A linear phase error will not distort the white-light signal (i.e., will not change its shape) but will cause an error in the measured surface height. Figure 3(b) shows that this effect is linearly proportional to amplitude Λ for small amplitudes and small duty cycles D. In this regime, the overall object profile will not be disturbed, but the apparent deflection will be underestimated.

Clearly performance of the stroboscopic interferometer is most closely related to the adjustable duty cycle D of the illumination. Figure 4 illustrates this point by presenting the fringe contrast $|u|$ loss [Fig. 4(a)] and the relative measurement error $\arg(u)$ $\lambda_0/4\pi\Lambda$ [Fig. 4(b)] as functions of the duty cycle.

4. Approximations at Short Pulse Widths

Clearly, small duty cycles are desirable, as we would have guessed from the beginning. In this limit, we can develop some rules of thumb regarding the expected performance of a stroboscopic white-light interferometer.

As we have seen, one of the undesirable effects of the object motion is the blurring of the interference fringes. This appears to be most severe when the vibratory motion is at its highest velocity, i.e., when the object is moving the furthest during the pulse width δt. From the derivative of ξ, the velocity is greatest (and >0) when

$$\varphi_0 = 0,\ 2\pi,\ 4\pi\ \ldots\ . \tag{13}$$

At these points in the vibration cycle, the second-order approximation

$$\sin(\varphi) \approx \varphi \tag{14}$$

is appropriate for all vibrational amplitudes, assuming once again a small pulse width δt. With this approximation,

$$u(K) = \frac{1}{2\pi D}\int_{-\pi D}^{\pi D} \exp(i\Lambda K\varphi)\mathrm{d}\varphi, \tag{15}$$

which readily evaluates to the sync function:

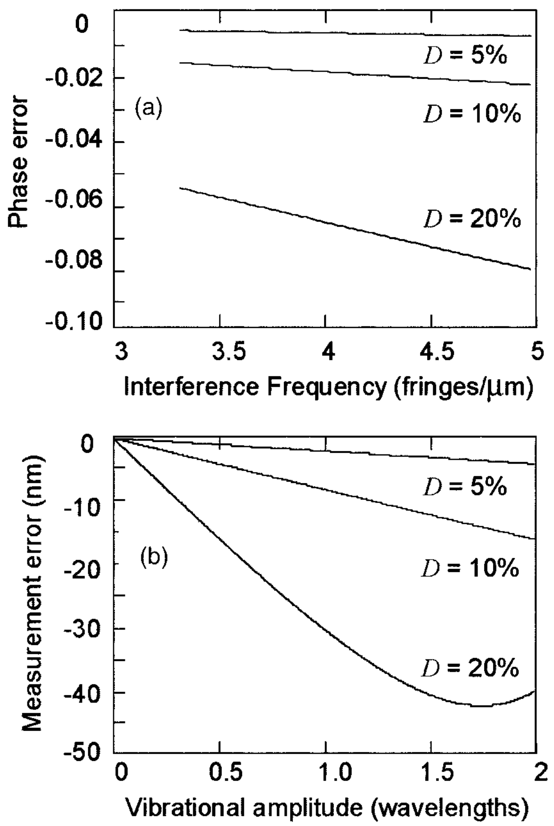

Fig. 3. (a) Dependence on interference spatial frequency K of the phase error (argument of u) attributable to stroboscopic interferometry for a vibrational amplitude of $\Lambda = 250$ nm and a vibrational phase of $\varphi_0 = \pi/2$. (b) Resultant measurement error as a function of vibrational amplitude.

$$u(K) \approx \frac{\sin(\Lambda K \pi D)}{\Lambda K \pi \mathrm{D}} \quad (\text{object deflection } \xi = 0). \tag{16}$$

The purely real result in expression (16) shows that at the high-velocity portions of the oscillation, the effect of the stroboscopic illumination is to decrease signal strength, particularly at the higher frequencies. Based on expression (16), to maintain a fringe contrast of >90%, the product of the peak-to-valley (P–V) vibrational deflection 2Λ of the object and the duty cycle D should be $<\lambda/25$.

The other extreme of the vibrational motion is a maximum deflection >0, which occurs at

$$2\pi ft = \pi/2,\ 5\pi/2,\ 9\pi/2\ \ldots. \tag{17}$$

Equation (10) becomes

$$u(K) = \frac{1}{2\pi D}\int_{-\pi D}^{\pi D} \exp[i\Lambda K \cos(\varphi)]\mathrm{d}\varphi. \tag{18}$$

This equation can also be evaluated by a second-order expansion, but a better approximation follows from the observation that the exponent in Eq. (18), in this case, is nearly constant for small φ, and therefore, we can account for the pulse width by treating the integrand of Eq. (18) as a constant

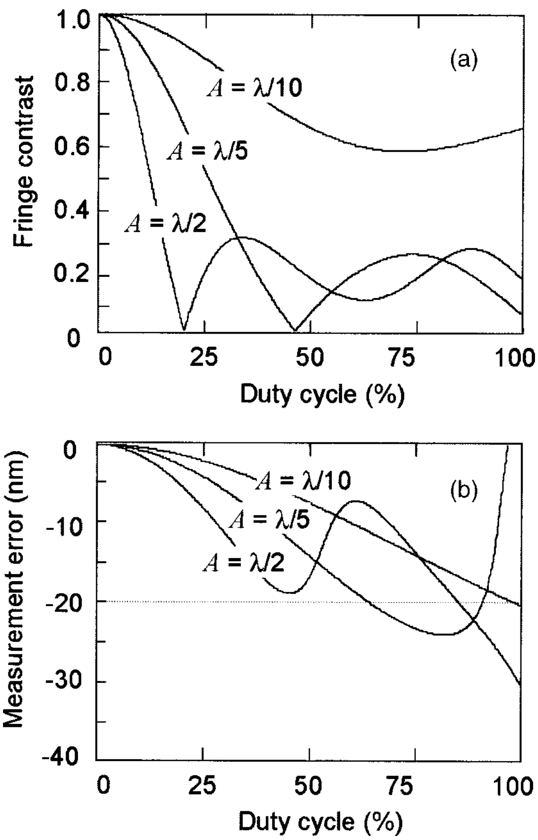

Fig. 4. (a) Dependence on duty cycle D of the fringe contrast at a vibrational phase of $\varphi_0 = 0$. (b) Dependence on D of the relative measurement error (which equals the error divided by the vibrational amplitude Λ) at a vibrational phase of $\varphi_0 = \pi/2$.

$$u(K) \approx \exp\left[i\Lambda K\, \overline{\cos(\varphi)}\right] \tag{19}$$

for an average cosine in the exponent

$$\overline{\cos(\varphi)} = \frac{1}{2\pi D}\int_{-\pi D}^{\pi D} \cos(\varphi)\mathrm{d}\varphi. \tag{20}$$

The averaged cosine evaluates once again to a sync function,

$$\overline{\cos(\varphi)} = \frac{\sin(\pi D)}{\pi D}, \tag{21}$$

so that in this limit there is a pure phase shift

$$u(K) \approx \exp\left[i\Lambda K \frac{\sin(\pi D)}{\pi D}\right] \quad (\text{object deflection } \xi \approx \Lambda). \tag{22}$$

Comparing this to the expected phase-shift ΛK at maximum deflection, we see that

$$\frac{\Delta\xi}{\xi} = \frac{\sin(\pi D)}{\pi D} \tag{23}$$

is the relative object position when the vibrating object reaches its maximum deflection.[11] Importantly, this relative underestimate $\Delta\xi/\xi$ is independent of ΛK. This means that in a scanning white-light interferometry system, there is no distortion of the white-light interference signal shape at the limit of $\xi \approx \Lambda$. However, the reported deflection is underestimated by an amount given by Eq. (23). As a rule of thumb, the relative error is less than 2% for a duty cycle of $D < 10\%$. Combining a $D < 10\%$ requirement with the 90% fringe contrast lower limit proposed in the paragraph following expression (16), the error is kept below 2 nm in all cases.

5. Conclusions

From these analyses, the following observations are relevant to using stroboscopic interferometry for out-of-plane displacements:

1. To avoid deleterious beating effects between free-running stroboscope pulses and the camera rate, the strobe frequency should be greater than the product of the frame rate and the number of camera gray levels. For a 60 Hz, 8-bit camera, this sets the minimum frequency to 16 kHz.
2. The key adjustable parameter for the instrument is the duty cycle, given by the product of the vibrational frequency f and the pulse width δt: $D = f\delta t$.
3. The duty cycle should be chosen to minimize the effect of fringe blurring. For most practical situations, we can estimate the fringe contrast relative to the nonstroboscopic case by means of the sync function in expression (16).
4. The deflection of the vibrating object will tend to be underestimated because of the time lag of the stroboscopic pulse. The measured deflection relative to the actual deflection may be estimated by the sync function Eq. (23).
5. It is feasible to correct for the measured deflection using a lookup table and thus allow for a larger duty cycle D, which may be necessary because of light-level considerations.
6. A duty cycle of less than 10%, the product of the P–V oscillation amplitude, and a duty cycle of less than wavelength/25 provide a fringe contrast of >90% and a measurement error of <2 nm.
7. Finally, because the fringe contrast is a wavelength-dependent phenomenon, it is best to either measure the wavelength directly or use a profiling algorithm that is insensitve to changes in the spectrum shape and centroid.

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