Lateral resolution and instrument transfer function as criteria for selecting surface metrology instruments

Xavier Colonna de Lega and Peter J. de Groot

Zygo Corporation, Laurel Brook Road, Middlefield, CT 06455
xcolonna@zygo.com and peterd@zygo.com

Abstract: We review definitions of optical resolution and how they relate to the Instrument Transfer Function of surface profiling interferometers. The corresponding *optical cutoff* provides a selection criterion for a given metrology application (PSD, waviness).

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1. Introduction

As manufacturers of optics and high-precision machined parts develop increasingly detailed specifications and manufacturing tolerances, they need to understand the limitations of the optical metrology instruments they select for process and quality control. The optical resolution of a surface profiler is in general object-dependent but it is possible to define regimes where the Instrument Transfer Function (ITF) and its optical cutoff capture key metrics about the capability of a given instrument. This paper reviews how the ITF relates to other optical transfer functions as well as traditional definitions of optical resolution.

2. Definitions of optical resolution and Optical Transfer Function

Conventional definitions of the resolution of an optical instrument relate to its ability to detect the presence of two adjacent source points. The Rayleigh criterion defines the minimum separation as the distance for which the first zero of the point spread function (PSF) of one source point overlaps the maximum of the PSF for the other. The Sparrow criterion is defined by the separation for which the resulting intensity pattern has no curvature in the center [1]. Approaching the definition of resolution from a different angle, the Abbe resolution limit is the largest diffraction grating pitch that cannot be detected by the optical system. The Rayleigh and Sparrow definitions correspond to objects that are Dirac distributions in the object plane while the Abbe definition corresponds to objects that are Dirac distributions in the spatial frequency domain. All are affected by the degree of coherence of the illuminating light, as shown in Table 1 where λ is the wavelength of light and NA is the image space numerical aperture of the optical system.

Table 1. Definitions of lateral resolution of an optical system as a function of the illumination type

Rayleigh (incoherent)	Sparrow (coherent)	Sparrow (incoherent)	Abbe (coherent)	Abbe (incoherent)
0.61 λ/NA	0.73 λ/NA	0.47 λ/NA	λ/NA	0.5 λ/NA

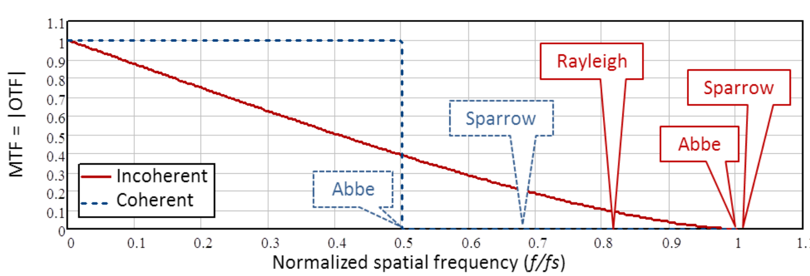

Fig. 1. Theoretical MTF of coherent and incoherent diffraction-limited imaging systems. Spatial frequencies are normalized to $fs = 2NA/\lambda$.

The Abbe resolution defines the spatial frequency cutoff of the optical system when the latter is considered to act as a linear filter on the image frequency content. Linear filtering actually only occurs under specific conditions of object illumination and imaging: low-NA and fully coherent or incoherent illumination. In the case of a coherent imaging instrument the optical system performs a linear filtering operation on the amplitude of the optical field, whereas it is the image intensity that is linearly filtered in an incoherent imaging system [2]. Practical optical systems rarely meet these exact criteria as illumination is in most cases partially coherent but linear filtering remains a useful approximation to understand the expected behavior of a given instrument.

The Optical Transfer Function (OTF) describes the frequency response of the filtering process [3]. Its magnitude, the Modulation Transfer Function (MTF) is shown in Fig. 1 for coherent and incoherent diffraction-limited optical systems. The figure also shows the spatial frequencies corresponding to sinusoidal object features having the same pitch as the Rayleigh and Sparrow separation limits.

3. Resolution in the context of surface height metrology: the Instrument Transfer Function

The carrier of information in an interferometer used for surface metrology is the phase of the light scattered by the object. The propagation of this information through the optical system can also be described as a linear filtering process under conditions more restrictive than for the filtering of amplitude or intensity. For a sinusoidal object of depth H and pitch $1/f$ illuminated by a plane wave along the surface normal (coherent illumination) the scalar complex amplitude takes the form of Eq. (1) in the plane of the object, where J_p is a Bessel function of the first kind of order p. We observe that the spatial frequency spectrum of the optical field consists of harmonics pf, corresponding to the diffraction orders of the grating. The suppression of some of these orders during the imaging process (diffraction angles outside of the imaging NA) prevents faithful reconstruction of the object amplitude, resulting in a distorted object profile. One cannot simply define a transfer function in this general case as the filtering process is intrinsically object-dependent.

$$U_g(x,y,0) = e^{i\frac{2\pi}{\lambda}H\cos(2\pi fx)} = \sum_{p=-\infty}^{+\infty} i^p J_p\left(\frac{2\pi}{\lambda}H\right)e^{ip2\pi fx} \tag{1}$$

$$U_g(x,y,0) \approx 1 + i\frac{2\pi}{\lambda}\frac{H}{2}\left(e^{i2\pi fx} + e^{-i2\pi fx}\right) \; for\; H \ll \frac{\lambda}{2\pi} \tag{2}$$

In the limit case where the object height is small compared to the wavelength of light the complex amplitude simplifies to Eq. (2). Coherent filtering by the instrument pupil will either pass the two diffraction orders (case where f is smaller than NA/λ) or clip them. The object amplitude is then perfectly reproduced in the image plane or any information about the presence of a grating is lost. In the first case, measuring the phase of the object amplitude, for example using a phase-shifting or spatial carrier technique [4], will then yield the expected surface profile. We observe that the ITF, which defines the linear filtering of the height information, is identical to the coherent OTF of the system in this small-object-height coherent regime [5].

There is a complementary regime where all energy-carrying diffraction orders fall within the cutoff of the filter function, in which case the object profile can also be faithfully reproduced. This allows relaxing somewhat the requirement of small height modulation for spatial frequencies much smaller than the cutoff.

For incoherent illumination we apply the formalism of Goodman [2], section 6.5.1, to show that the ITF is similarly given by the incoherent OTF. We first add a component $\exp(i\varphi)$ to U_g, which represents the contribution from the reference leg of the interferometer, and derive the spatial frequency spectrum G_g of the sum, see Eq. (3). We then compute the autocorrelation of G_g, drop the terms in $(H/\lambda)^2$, and multiply by the autocorrelation of the pupil function, which is by definition the OTF of the system; see Eq. (4) where we used the fact that $OTF(0) = 1$ and $OTF(f) = OTF(-f)$. A final Fourier transformation yields the intensity distribution in the image plane, Eq. (5), where we recognize a conventional two-wave interference pattern with a phase describing a sinusoidal surface height variation of depth $H \times OTF(f)$. We can thus identify the ITF as being identical to the incoherent OTF.

$$G_g(f_x,f_y) = \left(1+e^{i\varphi}\right)\delta(f_x,f_y) + i\frac{2\pi H}{\lambda\; 2}\left[\delta(f_x - f, f_y) + \delta(f_x + f, f_y)\right] \tag{3}$$

$$G_g \star G_g(f_x,f_y) \times OTF(f_x,f_y) = (2+2\cos\varphi)\delta(f_x,f_y) + \frac{2\pi}{\lambda}H\, OTF(f)\sin\varphi\left[\delta(f_x - f, f_y) + \delta(f_x + f, f_y)\right] \tag{4}$$

$$I_i(x,y,0) = 2 + 2\cos\varphi + \frac{2\pi}{\lambda}H\, OTF(f)\sin\varphi\cos 2\pi fx \approx 2 + 2\cos\left[\frac{2\pi}{\lambda}H\, OTF(f)\cos(2\pi fx) - \varphi\right] \tag{5}$$

Both coherent and incoherent results outlined above are valid within the approximation of small object heights and low NA (scalar diffraction). In practice, the ITF of a given instrument will deviate from the theoretical curves as a result of partial coherence (resulting for example from epi-illumination), aberrations, pupil apodization, detector sampling, etc. In this context, a pragmatic definition of the resolution limit of a surface profiler is the spatial frequency for which the measured ITF drops to 50%. This is the *optical cutoff* defined in ISO 25178-604 [6].

4. ITF and resolution as a criterion for selecting surface metrology instruments

The optical cutoff provides a convenient means of comparing the resolution of interferometric surface metrology tools. One should not *a priori* expect that two instruments that offer identical camera formats and fields of view also provide equivalent resolution. Their resolution will however be similar if their optical cutoffs are matched. Other criteria then come into play when evaluating the overall instrument metrology capability, such as noise floor, repeatability and reproducibility, coherent artifacts, vibration insensitivity, etc.

This optical cutoff is a primary selection criterion when matching the resolution of the instrument to a particular requirement, such as when characterizing the waviness of hard drive substrates or the Power Spectral Density of mirrors used for EUV lithography. Accurate knowledge of the ITF can further be used to correct the measured parameters in such applications.

To conclude, Fig. 3 plots the optical cutoff of the incoherent and coherent interference surface profilers shown in Fig. 4 as a function of part size. The curve covers almost four orders of magnitude along both axes, illustrating the flexibility and range of these instruments for characterizing critical surfaces with sub-Angstrom vertical resolution.

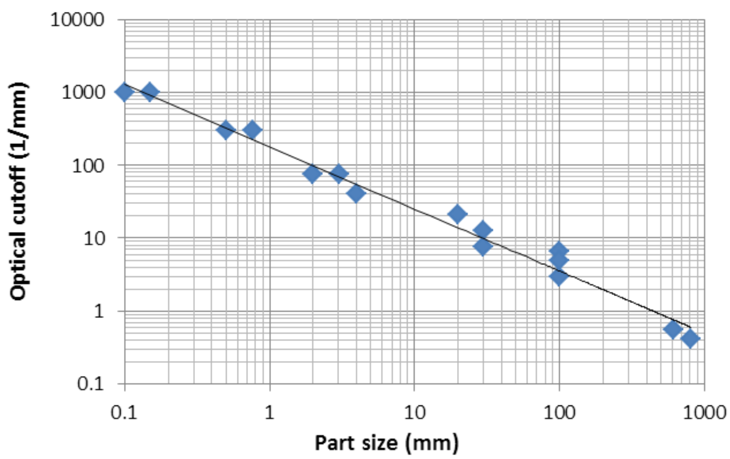

Fig. 3. Plot of the optical cutoff of a variety of interference surface profilers as a function of measured field of view.

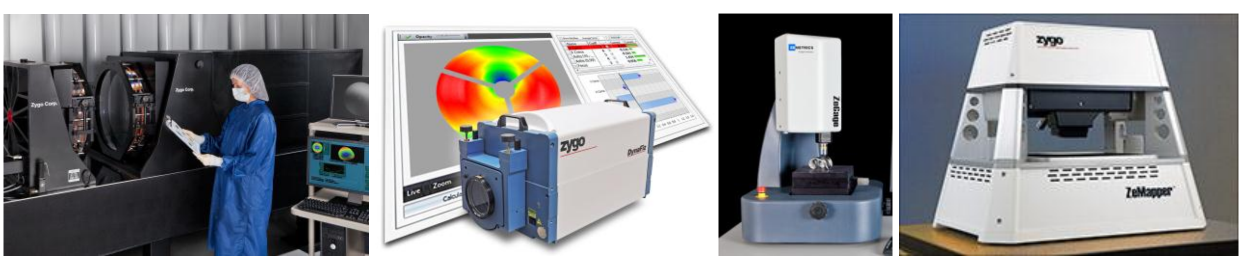

Fig. 4. Images of some of the instruments characterized in Fig.3.

5. References

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