Interpreting interferometric height measurements using the instrument transfer function

Peter de Groot and Xavier Colonna de Lega
Zygo Corporation
Laurel Brook Rd, Middlefield, CT 06455, USA

1 Introduction

Of the various ways of characterizing a system, one of the most appealing is the instrument transfer function or ITF. The ITF describes system response in terms of an input signal's frequency content. An every-day example is the graph of the response of an audio amplifier or media player to a range of sound frequencies.

It is natural therefore to characterize surface profiling interferometers according to their ITF. This is driven in part by developments in precision optics manufacturing, which increasingly tolerance components as a function of spatial frequency [1]. Metrology tools must faithfully detect polishing errors over a specified frequency range, and so we need to know how such tools respond as a function of lateral feature size.

Here we review the meaning, applicability, and calculation of the ITF for surface profiling interferometers. This review leads to useful rules of thumb as well as some cautions about what can happen when we apply the concept of a linear ITF to what is, fundamentally, a nonlinear system. Experimental techniques and example results complete the picture.

Our approach is informal, as is appropriate for a conference paper. The foundation for a rigorous understanding of the ITF is well documented in the literature, including the well-known books by Goodman [2].

2 Linear systems

ITF is most commonly understood to apply to linear systems, which share certain basic properties that lend themselves naturally to frequency analysis. Principally, the response of a linear system is the sum of the responses that each of the component signals would produce individually.

Thus if two frequency components are present in an input signal, we can propagate them separately and add up the results.

Another property of linear systems is that the response for a given spatial frequency f along a coordinate x is given by a corresponding ITF value characteristic of the system alone, independent of signal magnitude and phase. Thus to determine the output g' given an input g, we write

$$G'(f) = \mathrm{ITF}(f) \cdot G(f) \tag{1}$$

where

$$\begin{aligned} G(f) &= FT\{g(x)\} \\ G'(f) &= FT\{g'(x)\} \end{aligned} \tag{2}$$

and the Fourier Transform is defined by

$$FT\{\ \} = \int_{-\infty}^{\infty} \{\ \} \exp(-2\pi i f x)\, dx \,. \tag{3}$$

This is a powerful way of predicting system response to diverse stimuli.

3 OTF for optical imaging

A familiar ITF is the optical transfer function or OTF, which describes how an optical system reproduces images at various spatial frequencies. The modulus of the OTF is the modulation transfer function (MTF).

An approach to the OTF is to consider the effect of a limiting aperture in the pupil plane of an unaberrated imaging system. A plane wavefront generated by a point source illuminates a perfectly flat object (top left diagram in Figure 1). The object reflectivity profile may be dissected in terms of sinusoidal amplitude gratings over a range of spatial frequencies. Allowing each constituent grating its own DC offset, each grating generates three diffraction orders, -1, 0, 1. The separation of the ±1 orders in the pupil plane is proportional to the grating frequency.

According to the Abbé principle, if the pupil aperture captures all of the diffracted beams, then the system resolves the corresponding frequency. Assuming that the optical system is perfect and that it obeys the sine condition, the principle rays in Figure 1 show that the optical system faithfully reproduces the amplitude reflectivity frequency content up to a limiting frequency NA/λ. This *coherent imaging* MTF is therefore a simple rectangle, as shown in the top-right of Figure 1.

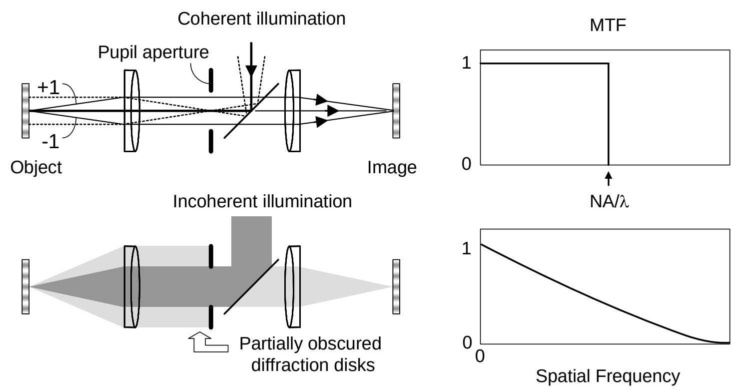

Figure 1: Illustration of incoherent and coherent light imaging systems (left) and the corresponding MTF curves (right).

The reasoning is much the same for an extended, incoherent source (lower left of Figure 1) [3]; although the results are very different. The various source points in the pupil generate overlapping, mutually-incoherent images that add together as *intensities*. As we move across the pupil, the obscurations of the ±1 diffraction orders vary. The calculation reduces to the autocorrelation of the pupil plane light distribution, which for a uniformly-filled disk is

$$\begin{aligned} \mathrm{MTF}(f) &= \tfrac{2}{\pi}\left[\phi - \cos(\phi)\sin(\phi)\right] \\ \phi &= \cos^{-1}(\lambda f / 2NA) \end{aligned} . \tag{4}$$

This curve, shown in the lower right of Figure 1, declines gradually from zero out to twice the coherent frequency limit. Incoherent imaging is often preferred in microscopes because of this higher frequency limit and softer transfer function, which suppresses ringing and other coherent artifacts.

Note that coherent systems are linear in *amplitude* and incoherent systems are linear in *intensity*. This leads to an ambiguity in the ITF for partially coherent light, addressed pragmatically by the *apparent* transfer function, which uses the ratio of the output and input modulations for single, isolated frequencies while simply ignoring spurious harmonics [4].

4 ITF for optical profilers

The ITF is so useful that it is tempting to use it even for systems that are explicitly nonlinear. Traditional tactile tools, for example, are nonlinear at high spatial frequencies because of the shape of the stylus; but their response is often plotted as a linear ITF [5]. If we are lucky, we find that over some limited range the system is satisfactorily approximated as linear. This is the case of optical profilers as well, with appropriate cautions.

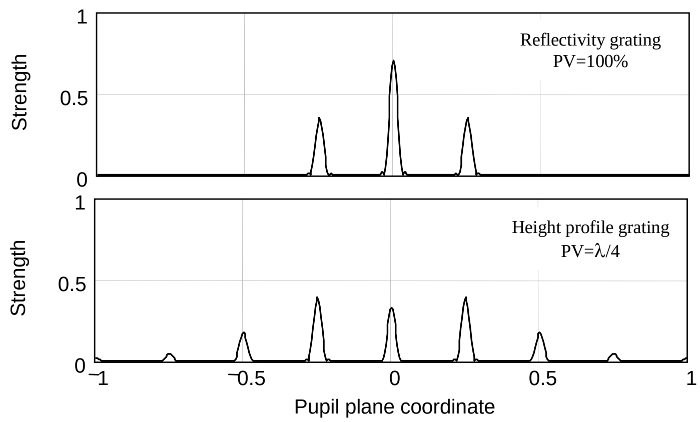

Figure 2: Comparison of the diffracted beams from *amplitude* (upper) and *phase* (lower) gratings illustrates the complex diffraction behavior of height objects, leading to nonlinear response when profiling surface heights.

Returning to the elementary concept of constituent gratings, consider coherent illumination of an object that has uniform reflectivity but a varying height. The surface impresses upon the incident wavefront a *phase* profile that propagates through the system to the image plane as a complex amplitude. Using any one of the known interferometric techniques, we can estimate the imaged phase profile and convert this back to height.

Just as before, a Fourier Transform of the object wavefront yields sinusoidal phase gratings over a range of spatial frequencies. Each grating generates diffracted beams, although Figure 2 shows that for phase gratings, the light spreads into higher angles than just the -1, 0, 1 orders present with amplitude gratings. Generally, the deeper the grating, the stronger and more numerous the higher diffraction orders, resulting in a very different situation from simple imaging. Spatial frequencies couple together, resulting in harmonics and beat signals in the imaged wavefront, inconsistent with the simple formula of Eq.(1). The response of the system is now inseparable from the nature of the object itself. Unavoidably, interferometers are nonlinear devices, as are all optical tools that encode height information as wavefront phase.

The solution to this dilemma is to restrict ourselves to small surface heights, where small means $<< \lambda/4$. For such small heights, diffraction from a phase grating is once again limited to the -1,0,1 orders and the higher orders become insignificant. The optical system responds to these small surface heights in much the same way as it images pure intensity objects, suggesting that we may be able to approximate the ITF by the OTF.

This last idea gains credence by considering a simple example. Arrange an interferometer so that the reference phase is balanced at the point where the intensity I is most sensitive to changes in surface height h. Then

$$I(h) = I_0 + I' \sin(kh) \tag{5}$$

where I_0 is the DC offset, I' is the amplitude of the intensity signal and $k = 2\pi/\lambda$. Inversion of Eq.(5) as the approximation

$$h \approx (I - I_0)/(I'k) \tag{6}$$

shows a linear relationship between height and intensity. More sophisticated algorithms will reduce in this limit to the same kind of simple linear equation. For a coherent system such as a laser Fizeau, the variation $(I - I_0)$ in Eq.(6) is proportional to the amplitude, since it is the product of the reference and object waves that gives rise to the measured intensity. For small surface heights, the coherent interferometer ITF is the same as the coherent imaging OTF. Similarly, for an incoherent system, we add together the interference intensity patterns for multiple source points—a calculation that mimics that of the incoherent imaging OTF.

To summarize the key conclusions of this section:

(1) The measurement of surface heights optically, e.g. by interferometry, is a fundamentally nonlinear process.

(2) A linear interferometer ITF is a reasonable approximation in the limit of very small surface deviations ($<< \lambda/4$).

(3) In the limit of small surface deviations, the interferometer ITF is the same as its imaging OTF.

5 Measuring interferometer ITF

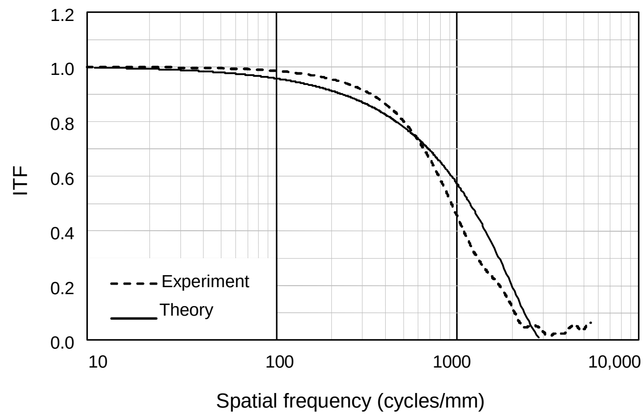

Figure 3: Comparison of the theoretical ITF magnitude (Eq.(4)) and experimental results for a white-light interference microscope using a 100X, 0.8 NA Mirau objective and incoherent illumination. The data derive from the profile of a 40-nm step object.

As a consequence of conclusion (3) above, it is sufficient to describe an interferometer's imaging properties to infer how it will respond to shallow height features. Of the many ways to measure OTF, one of the most convenient is to image a sharp reflectivity step [6], generated e.g. by depositing a thin layer ($<< \lambda/4$) of chrome over one-half of a flat glass plate. The idea is to determine the frequency content of the image via Fourier analysis and compare it to that of the original object. The ratio of the frequency components directly provides the OTF. The experiment does not require interferometry—we may even wish to block the reference beam to suppress interference effects.

Curiosity at least demands that we attempt the same experiment by directly profiling a step height [7]. The ITF in Figure 3 for one of our white light interferometers illustrates how closely the magnitude of the resulting experimental ITF magnitude matches the prediction based on the incoherent imaging MTF calculated from Eq.(4).

The resolution of low-magnification systems are often limited by the camera. Figure 4 shows the ITF of our laser Fizeau interferometer configured for coherent imaging. The coherent optical ITF is assumed equal to one for the theory curve over the full spatial frequency range shown, while the finite pixel size modulates the ITF by a sinc function.

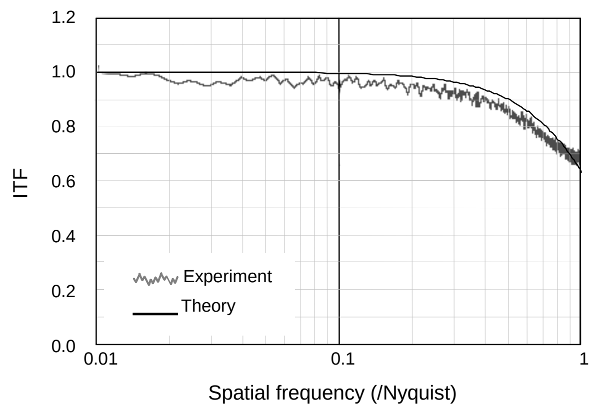

Figure 4: The predicted and experimental ITF curves for this 100-mm aperture coherent laser Fizeau interferometer are dominated by the lateral resolution of the 640X480 camera. Here the data stop at Nyquist because the sampling is too sparse above this frequency.

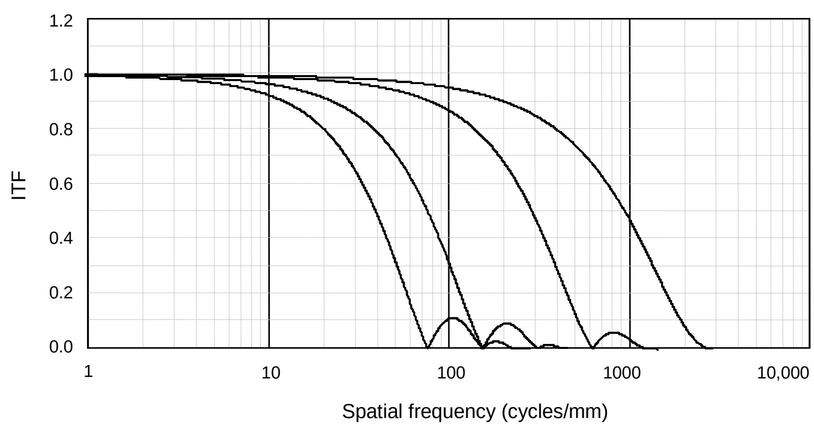

Figure 5: Theoretical ITF curves for 2.5X, 5X, 20X and 100X microscope objectives illustrate the spatial frequency overlap achieved in typical microscopes setups, and the influence of the camera at low magnification.

Figure 5 shows the coverage of a range of microscope objectives in incoherent imaging, including the effects of the camera. At lower magnifications, the lobes correspond to frequencies for which the optical resolution surpasses that of the camera. This figure illustrates how a range of objective on a turret provides complete coverage over a wide spatial frequency range.

6 Conclusions

Much of this paper has emphasized the precariousness of using a linear ITF for what is fundamentally a nonlinear process of encoding height into the phase of a complex wave amplitude. A more accurate model begins with an explicit calculation of this amplitude, then propagates the wave-front through the system to determine what the instrument will do.

Nonetheless, a kind of quasi-linear ITF is an increasingly common way to thumbnail the capabilities and limitations of interferometers in terms of lateral feature size, and to evaluate the effects of aberrations, coherence, defocus and diffraction [8]. As we have seen, the basic requirement for a meaningful application of a linear ITF is that the surface deviations be small. This allows us to *estimate* the expected behaviour for coherent illumination, as in laser Fizeau systems, and incoherent illumination, which is the norm for interference microscopes. Happily, in this limit of small departures, the profiling behaviour follows closely that of imaging, so that with appropriate cautions we can get a good idea of expected performance using the imaging OTF as a guide to the expected ITF.

References and notes

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