Displacement Measuring Interferometry

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4.1 Introduction

The wavelength of light provides an exceedingly precise measure of distance and is the foundation for commercial interferometric measurement tools that monitor object positions with a resolution better than 1 nm for objects traveling at 2 m/s. A wide range of applications include machine tool stage positioning and distance monitoring over length scales from a few millimeters to hundreds of kilometers in space-based systems.

Displacement measuring interferometry or DMI enjoys multiple advantages with respect to other methods of position monitoring. In addition to high resolution, wide measurement range, and fast response, the laser beam for a DMI is a virtual axis of measurement that can pass directly through the measurement point of interest (POI) to eliminate Abbe offset errors. Figure 4.1 illustrates the position of DMI in terms of resolution and dynamic range with respect to capacitive gaging, optical encoders, and linear variable differential transformer (LVDT) methods. The measurement is noncontact and directly traceable to the unit of length. Since the first practical demonstration of automated, submicron stage control using displacement interferometry in the 1950s, DMI has played a dominant role in high-precision positioning systems.

This chapter is intended as an overview of the current state of the art in DMI as represented by the technical and patent literature. The chapter structure is correspondingly encyclopedic and allows for access to specific topics without necessarily reading the chapter linearly from start to finish.

Following this brief introduction, we begin in Section 4.2 with fundamental physical principles of DMI, followed in Section 4.3 by a review of phase detection methods most
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common in practical implementations, introducing the essential concepts of homodyne and heterodyne detection. Section 4.4 considers ways of generating wavelength-stable light having the coherence and modulation characteristics essential for heterodyne DMI. Section 4.5 catalogs some of the more common optical interferometer configurations sensitive to various measurement parameters such as displacement, angle, and refractive index. With these tools in hand, we then examine in Section 4.6 the question of system performance in terms of the measurement uncertainty, encompassing error sources such as wavelength instability, Abbe offset error, cyclic error, air turbulence, and thermal drift. A gallery of practical DMI applications follows in Section 4.7, considering calibration and validation tasks as well as integration of DMI into a complete machine for continuous motion control and/or measurement. The applications section includes examples of unusual technology approaches that may find more common usage going forward.

4.2 Fundamentals

The high precision of DMI leverages the rapid change of phase as light propagates, equivalent to a \(2\pi\) phase shift for a distance of less than half a micron for visible light. This fine-scale, traceable metric is accessible through interferometric methods of comparing reference and measurement beams generated from a common source light beam and then recombined.

To establish terminology and notation, we provide here a mathematical description of the principles of the technique. The oscillating electric field of a source beam of amplitude \(E_0\) is

\[
E(t, z) = E_0 \exp \left[ 2\pi i \left( ft - \frac{nz}{\lambda} \right) \right] \tag{4.1}
\]

where

- \(f\) is the frequency of oscillation
- \(\lambda\) is the vacuum wavelength
- \(z\) is the physical path length
- \(n\) is the index of refraction

The frequency at visible wavelengths is very high—approximately \(6 \times 10^{14}\) Hz—making it difficult to detect the phase \(2\pi (ft - nz/\lambda)\) of \(E(t, z)\) directly. To access the wavelength as a unit of measurement, we need to remove or at least drastically slow down the optical frequency component. This is why we use interferometry.

Referring to Figure 4.2, the nonpolarizing beam splitter (NPBS) splits the source light into two beams, labeled 1 and 2 for the measurement and reference beams, respectively. These beams follow different paths and therefore have different phase offsets related to the propagation term \(nz/\lambda\):

\[
E_1(t, z_1) = \eta E_0 \exp(2\pi i ft) \exp \left( -\frac{2\pi i nz_1}{\lambda} \right) \tag{4.2}
\]
where 

\[ z_1, z_2 \text{ are the path lengths traversed by beams from the point that they are separated by the NPBS to the point that they are recombined} \]

\[ r_1, r_2 \text{ are their relative strengths with respect to the original complex amplitude } |E_0| \]

When the two beams superimpose coherently on a square-law detector, the time average of the resulting intensity is 

\[ I(z_1, z_2) = |E_0|^2 \left( \left| \exp(2\pi ift) \right|^2 \right) r_1 \exp \left( -\frac{2\pi i n z_1}{\lambda} \right) + r_2 \exp \left( -\frac{2\pi i n z_2}{\lambda} \right) \] (4.4)

The frequency term \( ift \) averaged over time becomes a constant:

\[ \left< \left| \exp(2\pi ift) \right|^2 \right> = 1 \] (4.5)

but the final expression preserves the optical path difference \( z_1 - z_2 \): 

\[ I(L) = I_1 + I_2 + \sqrt{I_1 I_2} \cos[\phi(L)] \] (4.6)

where 

\[ I_1 = |r_1 E_0|^2 \] (4.7) 
\[ I_2 = |r_2 E_0|^2 \] (4.8)
the beat frequency between the two modes,\(^3\) obtained by arranging for the two beams to interfere by means of a linear polarizer prior to directing them onto a single detector. Locating the minimum in the beat frequency as function of resonator length controls the resonator length.

As shown in Figure 4.10 by the dashed lines, the laser output for stabilization can be the weaker beam that exits the rear of the tube, thereby making all of the main output of the laser available to the metrology application. A recent design exploits the elliptically polarized beams that result from the inherent anisotropy in the laser tube to produce the feedback signal without any additional polarization optics.\(^39,40\) The light exiting the source transforms into two mutually orthogonal linearly polarized beams (at two slightly different frequencies) by means of a QWP. In this type of laser, the production of the two frequencies for heterodyne DMI is integral to the stabilization scheme.

While the Zeeman method has some drawbacks from a control standpoint,\(^41\) Hewlett-Packard (now Agilent),\(^31,33\) Zygo Corporation,\(^42\) and other manufacturers produce Zeeman laser DMI systems that have played a significant and widespread role in metrology. A drawback of this method is the low heterodyne frequency and the resulting limitation on the maximum slew rate of the target. A further drawback is the requirement on single-mode operation of the tube,\(^43,44\) which limits the length of the laser tube to less than about 10 cm and in turn reduces the achievable output power compared to the longer tubes used in polarization-stabilized lasers.

The quality of the exiting beams in terms of the orthogonality and ellipticity are key parameters that have an impact on the cyclic errors, as described in Section 4.6. Early characterizations of these parameters report deviations from orthogonality of \(4^\circ–7^\circ,45\) while more recent measurements suggest a much smaller value of \(0.3^\circ\) and an ellipticity of 1:170 in the electric field strength.\(^46\)

### 4.4.2 Polarization-Stabilized Lasers

In contrast to the Zeeman-stabilized system, polarization-stabilized DMI laser source systems separate the stabilization and frequency shifting functions.\(^17,47–49\) The stabilization technique relies on matching the intensities of two adjacent longitudinal modes as shown in Figure 4.11.\(^50,51\) The presence of two orthogonally polarized modes under the gain curve

![FIGURE 4.11](image)

Two-mode operation of a polarization-stabilized laser.
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makes it possible for the tube in a polarization-stabilized laser to be longer than in the Zeeman-stabilized laser (~30 cm).

A BS as shown in Figure 4.12 samples a small portion of the output beam. A Wollaston prism (or other suitable birefringent prism) divides the sampled beam according to the two orthogonal polarizations corresponding to the two lasing modes and directs them on to two detectors $D_1$ and $D_2$. The control system strives to minimize the difference in intensity observed at the two detectors $D_1$ and $D_2$ by adjusting the cavity length. As shown in Figure 4.11, which depicts the relative intensities of the modes when the laser is in the stabilized condition, the two modes are disposed symmetrically with respect to the wavelength corresponding to the peak of the gain curve.

The polarizer following the BS passes only one of the modes, which then passes through an acousto-optic modulator (AOM) to produce the two mutually orthogonal linearly polarized frequency-shifted beams arranged to overlap and emerge parallel to one another from the laser by using a second birefringent prism (not shown). The frequency difference $\Delta f$ corresponds to the drive frequency of the AOM. Although one of the modes is rejected, the increase in tube length (when compared to a Zeeman-stabilized laser) for two-mode operation more than makes up for the loss of one of the modes, resulting in a net gain in output power. This method of frequency shifting allows for large frequency differences relative to the Zeeman frequency split, with a 20 MHz frequency difference being common, which supports much higher slew rates of the target. This technique is however less efficient in its use of the available light. One way to recover the lost power is to use the two lasing modes to supply two independent interferometers.

Commercial-stabilized laser sources typically specify a relative “vacuum wavelength accuracy” or “unit-to-unit variability” of $\pm 0.1\text{--}0.8 \times 10^{-6}$. These numbers can be the basis for the estimation of a standard uncertainty in the vacuum wavelength. If a more accurate value of the wavelength is desired, the wavelength should be determined for the source in question by comparison against an iodine-stabilized He-Ne laser or against a frequency comb. Stabilities for commercial sources are typically specified over various time scales, with short-term (1 h), medium-term (24 h), and long-term (over the laser lifetime) relative wavelength stabilities of $\pm 0.5\text{--}2 \times 10^{-9}$, $\pm 1\text{--}10 \times 10^{-9}$, and $\pm 10\text{--}20 \times 10^{-9}$, respectively. A recent report on the measurement of 28 different laser heads of both types by
For dual-axis motion as required by an x–y stage, the preferred object is a plane mirror rather than a retroreflector. Figure 4.15 shows how this may be accomplished using a QWP. The QWP converts linear polarization to circular and then back to an orthogonal linear polarization after reflection from the object mirror, which is now free to move orthogonal to the line of sight without disturbing the beam paths. Figure 4.16 shows how the double pass to the object mirror with a retroreflection compensates for a tilt $\theta$ of the mirror. An additional benefit of the double pass is a finer measurement resolution, with one full $2\pi$ phase cycle for every quarter wavelength of object motion.

Another popular design for a plane-mirror interferometer (PMI) is the high-stability type or high-stability plane-mirror interferometer (HSPMI), shown in Figure 4.17. This geometry self-compensates for any changes in the interferometer optics—for example, thermal expansion—by configuring the reference and measurement paths symmetrically,
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with the same amount of glass in each beam. This approach brings thermal sensitivity to below 20 nm/°C in well-designed HSPMI packages.

Although shown as 2D in the figures to clarify their function, actual interferometers are 3D and have more complicated beam paths, as shown in Figure 4.18. Practical designs use optics of BK7 or crystalline quartz, vacuum-grade low-volatility adhesives, and stainless steel housings. Although there are a large number of reflections and transmissions through various optical surfaces, the net light efficiency of a commercial HSPMI is approximately 60%.

**FIGURE 4.17**
High-stability plane-mirror interferometer (HSPMI).

**FIGURE 4.18**
HSPMI and corresponding beam paths presented in three dimensions.
Stage metrology in particular often requires measurements of multiple degrees of freedom (DOF), including stage pitch and yaw. For this reason, a variety of angle measurement interferometers have been developed, often using the HSPMI as a building block. Figure 4.19 illustrates one way to achieve this by integrating two HSPMI subsystems into a single, partially monolithic system with high thermal and mechanical stability.96 Here the distance measurement involves only the upper pair of beam paths to the object mirror, whereas the angle measurement involves both upper and lower paths to the object, with the roles of measurement and reference beams (labeled 1 and 2, respectively) reversed between the two pairs. Figure 4.20 shows one implementation using cemented bulk optical components and intended for a 3 mm beam diameter. Multiaxis interferometer systems can be quite complex, including half a dozen distinct motion measurements, often referenced to different parts of a mechanical system or metrology frame, with individualized beam steering to compensate for any imperfections in the optical components.

In addition to the basic metrology function of interferometer optics, it is common to use interferometers to perform the auxiliary function of tracking variations in the effective wavelength $\lambda/n$ in the ambient medium that surrounds the DMI system. The differential plane-mirror interferometer (DPMI) as shown in Figures 4.21 and 4.22 has the ideal geometry for a wavelength tracker, reporting variations in the measured path $(n-1)L$ and hence the index and effective wavelength.1797

Figure 4.23 illustrates one way to measure motions orthogonal to the nominal target motion, for example, to monitor the straightness of travel of a stage.98 In this case, the interferometer is measuring the lateral displacement of the birefringent prism, which is mounted to the moving stage. The straightness deviation $\Delta x$ is detected as a distance change given by

$$\Delta L = 2\sin(\gamma)\Delta x$$

(4.16)
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Category 2—uncertainty from sources that affect the measurand but in fact are not directly attributable to it, that is, from spurious displacements. These contributors are represented by \( \Delta \phi \) and the second term of Equation 4.18, and unlike the contributors in Category 1, the magnitude of these contributions is not dependent on \( D \).

Table 4.2 is a nonexhaustive list of common contributors in each of the earlier categories. Many of the contributors common to other measurement techniques (e.g., cosine error and Abbe error) are included here clarify their interpretation in the context of interferometric displacement measurement. Some uncertainty sources that are common to any measurement technique, for example, thermal changes in the metrology frame, are not discussed here but would contribute as they would in any measurement.

### 4.6.2 Vacuum Wavelength

The vacuum wavelength \( \lambda \) establishes the linkage between the unit of length and the change in phase, and any associated uncertainty \( u(\lambda) \) contributes directly in the calculated displacement. The uncertainty in the measurand \( u_\lambda(D) \) depends on the relative uncertainty in the wavelength \( u(\lambda)/\lambda \) and is proportional to the measurand as given by\(^{114,115}\)

\[
u_\lambda(D) = \frac{u(\lambda)}{\lambda} D
\]  

(4.19)

Uncertainty in the vacuum wavelength \( \lambda \) has multiple contributors: the uncertainty in the determination of the absolute value of the wavelength and a contribution arising from short- and long-term drift\(^{116,117}\). Displacement interferometers typically use stabilized He-Ne lasers emitting at 633 nm as the light source, and the uncertainty in the wavelength and the associated stability depends on the method of stabilization as summarized in Table 4.1. For applications demanding a lower uncertainty in the vacuum wavelength, a reference laser such as an iodine-stabilized laser may be included within the system and used to monitor the wavelength of the metrology laser used for interferometry\(^{90,90}\).

In the case of two-frequency laser sources intended for use in heterodyne systems, a rather subtle error can result from the slightly different nominal vacuum wavelength for the two beams of slightly different frequencies (and different polarizations). The difference depends on the frequency difference (or split frequency). In such systems, it is important...
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The equation also gives (in round numbers) the required changes in key environmental parameters to produce a $1 \times 10^{-6}$ change in the index in more commonly used units. The refractive index is most sensitive to changes in temperature and pressure with relatively large changes in humidity being required to cause a comparable change in the index. Note that the coefficients are signed, and while this usually is not of consequence in the determination of the uncertainty, it is critical in establishing the sign of the error. The associated uncertainty $u(n_{air})$ is a function of the uncertainties in the temperature $u(T)$, pressure $u(P)$, and humidity $u(H)$ and is given by

$$u(n_{air}) = \sqrt{K_T^2 u^2(T) + K_P^2 u^2(P) + K_H^2 u^2(H)}$$  \hspace{1cm} (4.22)

There is also an “intrinsic” uncertainty associated with this empirical expression that derives from the uncertainty in the data used to derive them. In other words, even if the inputs to the equation are known exactly, there is an uncertainty in the calculated index that results from the uncertainty in the coefficients of the modified Edlén equation. The developers of the equation estimate this uncertainty contribution to be $3 \times 10^{-8}(3\sigma)$, or a standard uncertainty of $1 \times 10^{-8}$ or 10 parts per billion (ppb), corresponding to 10 nm in a displacement over 1 m. This intrinsic uncertainty should be included in determinations of $u(n_{air})$. In almost all practical applications, contributions from the uncertainties associated with the determination of the input parameters dwarf this contribution.

The weather station method depends on fixed assumptions regarding the composition, potentially leading to an important uncertainty in the calculated index. The modified Edlén assumes a CO$_2$ concentration of $450 \times 10^{-6}$. The CO$_2$ concentration can vary—a prime example is a higher than assumed concentrations of CO$_2$ caused by human respiration. Fortunately, changes in the index of $\sim 2 \times 10^{-8}$ require a change in CO$_2$ concentration of $150 \times 10^{-6}$ and only become significant in the most demanding applications. A more significant contribution is due to changes in composition due to the presence of hydrocarbons, for example, acetone, which causes an index change of $10^{-7}$ for a contamination level of $130 \times 10^{-6}$. Solvent vapors are present in many metrology environments, for example, the metrology of optics, where solvents such as acetone, alcohol, and various other volatile solvents are often used for cleaning, resulting in marked deviations from the assumed composition. The composition of the air is not typically monitored in such environments, although in general the presence of a detectable odor is a good indication of the presence of a volatile hydrocarbon at levels that are significant for the highest accuracy measurements. One way this contribution may be incorporated into Equation 4.22 is via an additional term that captures the uncertainty in the concentration of the particular hydrocarbon and the appropriate sensitivity.

For high-accuracy determinations of the index, great care is required in making measurements of temperature, pressure, and humidity, as described in the classic paper by Estler. The measurements of the environmental parameters now also become part of the traceability chain. Recent refractometer comparisons have shown that it is possible with careful measurements to reduce the contribution from uncertainties in the input parameters to the point where the uncertainties in the parameters of the modified Edlén equation are comparable or even dominate. This however requires extremely careful measurements of the input parameters, something that is typically not easily achieved in nonlaboratory measurements. Factors such as the location of the sensors, gradients in temperature and pressure, thermal inertia, and self-heating of sensors also complicate the measurement of environmental parameters.
of the reticle and wafer stage to nanometer levels to achieve the stringent overlay requirements.\textsuperscript{214} Displacement measurements to the required accuracy demand exceptional attention to detail and several advanced features, some of which are discussed in the following.

Although the basic interferometer arrangement in microlithography systems (Figure 4.35) measures displacements and rotations in the plane of the wafer and reticle,\textsuperscript{183,215} modern tools measure and control all the DOF of these stages. Additional measurement axes compensate for Abbe offsets and for numerous other measurements and require in excess of 50 channels of displacement metrology per exposure tool. Advances in the signal processing electronics and the low noise floor intrinsic to the heterodyne process make it possible to power multiple measurement channels with subnanometer noise performance from one laser head. These systems also may use specialized interferometers to reject any structural deformations by making a differential measurement between the wafer or mask and the projection or inspection optics.\textsuperscript{216,217} Additionally, the deviation from flatness resulting from fabrication and mounting of the target mirrors becomes significant, requiring characterization of mirror shape. Characterization must be carried out in situ to measure the as-mounted mirror figure, by comparison to an external straightedge\textsuperscript{190} or by making multiple redundant measurements of the mirror using multiple interferometers.\textsuperscript{192,193} Similarly, there is a need to characterize deviations from squareness in the reference mirrors and the motion axes. This is typically performed in situ using reversal techniques.\textsuperscript{218,219}

The high velocities and requirements on synchronized motion in microlithography impose stringent requirements in the acceptable variation in the data age, as detailed in Section 4.7. The data age uncertainty results in a positioning error proportional to the velocity, and modern measurement electronics are designed with the ability to adjust the data age in order to minimize or eliminate the data age difference.\textsuperscript{18,220}

Modern lithography tools may use a combination of optical encoders (see Section 4.8) and conventional interferometers to overcome air turbulence. These two systems complement each other: The short air path typical of encoders minimizes the effects of air turbulence, thus realizing an improvement in short-term repeatability,\textsuperscript{150} while the conventional interferometer provides superior linearity when compared to the nonlinearities encountered in
A combination of plane mirrors and corner cubes have been used for large ranges of motion. Murty introduces an additional plane mirror into the measurement beam path, which retroreflects the output beam of the corner cube for a second trip through the corner cube before it returns to the interferometer (Figure 4.37d). This arrangement makes the system shear free but requires an additional component whose stability, alignment, and figure now directly influence the measurement.

### 4.7.3 Measuring Machines

Interferometers serve as the primary metrology in a number of high-performance CMMs. The term CMM is used rather loosely here and encompasses measuring machines of all kinds ranging from conventional multiaxis CMMs and some more unusual multipurpose CMMs to specialized machines such as metrological atomic force microscopes (AFM), line scale comparators, and devices for the evaluation of encoders.

The use of interferometers in CMMs is characteristic of the highest accuracy machines. One example is the Moore M48 CMM at the National Institute of Standards and Technology (NIST). The configuration of this machine is conventional (see Figure 4.38a), the exceptional performance attributable to the fidelity of the mechanical motions, careful error mapping, and stringent conditioning of the environment. Other machines rely less on the fidelity of the mechanical motions and take a different approach to mitigating the effect of the Abbe offset. Some rely on a combination of an adherence to the Abbe principle and active cancelation of angular error motions. Others locate the

![FIGURE 4.38](image)

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The diameter is determined by comparing the part under test to a length artifact that serves as a reference for the displacement interferometers.

DMIs also find application in high-accuracy instruments for the measurement of ring and plug gages and in more specialized instruments for the calibration of pressure standards. DMIs offer a combination of features that are important to such measurements, especially at the NMI level, that is, direct traceability to the unit of length and high resolution over a long measurement range. The last attribute makes it possible to use such instruments in comparator mode between artifacts of widely varying dimensions.

DMIs are common in metrological scanned probe microscopes to characterize standard artifacts for calibration of other instruments such as critical dimension scanning electron microscopes (CD-SEM) and other scanned probe instruments. This class of machines is exemplified by the NIST molecular measuring machine (M3), pictured in Figure 4.41. Figure 4.42 shows an exploded view of one of the interferometers, the x-axis interferometer, which measures the displacement of the probe tip relative to the metrology mirrors that form the box that carries the sample. These instruments use DMIs for minimizing Abbe offset errors and for direct traceability to the unit of length. While an ideal arrangement would make a differential measurement between the sample and the probe tip, the small size of the tip and other practical considerations make this extremely difficult. Therefore, virtually all such instruments make a measurement of the displacement of the structure that supports the tip. Interferometer geometries differ, some being designed to make a differential measurement between the sample holder and the tip holder.

FIGURE 4.41
devices so as to reject uncertainty contributions from angular error motions of the target. Synchronization of the data acquisition from the two sensors is important and is especially critical at high target velocities or in dynamic situations.

Gravimeters use DMIs for the determinations of the absolute value of the acceleration due to gravity. These devices are Michelson interferometers in which the measurement target is part of a free-falling mass within a vacuum chamber and an inertial reference carries the reference mirror. The vacuum mitigates both the effects of atmospheric drag and the uncertainty contributions due to index. Gravimeters require a length standard known to an uncertainty better than one part in $10^9$ that is typically provided by an iodine-stabilized laser, although another stabilized laser may be used with periodic recalibration.

Primary pressure standards use interferometric measurement of the differential displacement of the liquid columns of manometers, either directly by reflecting the measurement beam from the Hg surface or by using a corner cube suspended in the mercury on a suitable float.

### 4.8 Alternative Technologies

#### 4.8.1 Absolute Distance Interferometry

DMI according to the definition in the introductory part of this chapter refers to measurements of displacements or changes in position. A DMI reports only how object positions change while being measured, not how far away they are from a specific reference point in space. In a conventional DMI, if the measurement beam is blocked and subsequently re-established, there is no information about any change in position of the object that occurred while the beam was blocked.

For many applications, the distance from the object to a reference position is of importance and needs to be measured at any given moment in time, without relying on a continuous time history of the object motion. Instruments for these applications measure the absolute distance $L$ as opposed to a relative displacement $D = L_f - L_i$ from a first position $L_i$ to a second position $L_f$. There is abundant literature on this topic representing a wide range of solutions. Setting aside ranging systems that operate by pulsed time of flight or microwave intensity modulation, the majority of coherent or interferometric systems for absolute distance measurement employ multiple or swept-wavelength sources.

Multiple-wavelength methods have roots in the earliest interferometers for length standards. The principle of measurement resides in the dependence of the interferometric phase on wavelength. Recalling Equation 4.9

$$\phi(L) = \left(4\pi n \frac{L}{\lambda}\right)$$

(4.42)

it is clear that there is a linear relationship between the phase $\phi$ and the angular wave number $\sigma$, where

$$\sigma = \left(4\pi n \frac{1}{\lambda}\right)$$

(4.43)
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The rate of change of phase with wave number $\sigma$ is proportional to the difference $nL$ in the optical lengths for the measurement and reference paths. The historical method of excess fractions relies on matching up phase values for a sequence of discrete wavelengths to a specific distance using tables or a special slide rule. An alternative methodology involves the concept of an equivalent or synthetic wavelength $\Lambda$ for a pair of wavelengths $\lambda_1 > \lambda_2$ and a synthetic phase $\Phi(L)$ calculated from the difference in the corresponding phase measurements $\phi_1, \phi_2$:

$$\Phi(L) = \frac{4\pi nL}{\Lambda}$$  \hspace{1cm} (4.44)

$$\Lambda = \frac{\lambda_1 \lambda_2}{(\lambda_1 - \lambda_2)}$$  \hspace{1cm} (4.45)

$$\Phi = \phi_2 - \phi_1$$  \hspace{1cm} (4.46)

For the simplest case of two wavelengths, the absolute measurement is unambiguous over a range defined by at least half the synthetic wavelength. Multiple-wavelength emissions from CO₂ and other gas lasers are a natural choice for this type of absolute distance interferometry. Two or more single-mode lasers locked to a Fabry–Pérot etalon, as illustrated in Figure 4.48, provide high relative wavelength stability for large variations in optical path length. A simple and compact configuration for short-path differences is one or more multimode laser diodes. Most recently, work has been carried out to take advantage of

**FIGURE 4.48**
comb spectra from frequency comb lasers\textsuperscript{339,340} and to optimize the choice of wavelength for the largest possible unambiguous range.\textsuperscript{341} An alternative approach to multiple wavelengths is a continuously swept wavelength, sometimes referred to as frequency-modulated continuous wave (FMCW) ranging. Just as in multiple-wavelength methods, the principle follows the observation that the interference phase varies linearly with wave number defined in Equation 4.43 at a rate proportional to the distance:

\[
\frac{d\phi}{d\sigma} = 2\pi nL \tag{4.47}
\]

A linear variation of laser wave number generates an interference signal having a frequency that is also linearly dependent on the distance \( L \), with no limit to the available unambiguous range, provided that the source is sufficiently coherent. Laser diodes have dominated this technique as tunable sources for the past two decades.\textsuperscript{342,343} Most often, the available tuning range is insufficient to resolve the absolute distance to within a wavelength, and consequently simple systems for FMCW ranging are usually not capable of the same precision as DMI. High precision on the order of one part in \( 10^9 \) of the measured distance is however feasible with advanced sources and sufficient care.\textsuperscript{344}

### 4.8.2 Optical Feedback Interferometry

So far, we have considered interferometer geometries in which splitting the source light into reference and measurement beams followed by recombination at a detector establishes the interference effect. However, it was discovered early in the history of the laser that reflected light directed into the laser cavity would produce wavelength and intensity modulations that could be used directly for distance and velocity measurement.\textsuperscript{345} The basic geometry for such systems can be very simple: All that is required is a path for reflected light to enter the laser—a condition that is almost unavoidable in many cases—and a detector for observing the modulations in the laser output in response to the phase of this reflected light.

The optical feedback or self-mixing effect is particularly strong in semiconductor laser diodes because of the strong gain medium and weak front-surface reflection of the laser cavity, which allows even weakly reflected light to influence the behavior of the laser. The amount of feedback need not be high for the effects to be significant—feedback or backscattered light at levels as small as \( 10^{-9} \) times the emission intensity is sufficient to measurably alter the power output and frequency of the laser. Figure 4.49 shows a simple system for velocimetry, where the detection is either the oscillation in the laser optical power output or even more simply the variation of the terminal voltage or driving current of the laser itself.\textsuperscript{346}

The principles of optical self-mixing in lasers have been extensively analyzed, with most physical models based on the inclusion of the object itself as part of the laser cavity.\textsuperscript{347} The additional reflecting surface modulates the threshold gain of the system. A characteristic of self-mixing interferometry is that the signal shape is not sinusoidal but is rather more like a saw tooth, as a result of the laser system phase lock to the external reflection, providing a directional discrimination to the homodyne signal without the need for phase shifting.\textsuperscript{348} The phase locking also introduces a frequency shift to the laser output, providing another method of detection that in multimode lasers can be particularly effective.\textsuperscript{349,350}
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Frequencies must be carefully preserved, which may require using two fibers in place of one in a separated beam delivery or providing a frequency shifter local to the interferometer itself (Figure 4.52). Finally, an additional detector must be placed at each interferometer of a heterodyne system to monitor the relative phase of the two-frequency source beams, which otherwise would become indeterminate while traveling through fibers. In some proposed systems, this last requirement is satisfied by a push–pull interferometer design such as the one shown in Figure 4.53 in which both of the source beams pass through the measurement and reference paths together. An advantage of this push–pull approach is that it effectively doubles the resolution of the measurement.

FIGURE 4.51
Dispersion interferometry for correction of air turbulence to the nm level. (From Deck, L.L., Dispersion interferometry using a doubled HeNe laser, Zygo Corporation, Middlefield, CT, Unpublished, 1999.)

FIGURE 4.52
Fiber-coupled light source with remote frequency shifters. (Photo courtesy of Zygo Corporation, Middlefield, CT.)
An entirely different use of optical fibers in displacement interferometry relies on precision measurements of the fiber length itself, in an area broadly categorized as fiber sensing. In these instruments, the distance traveled by the light is proportional to a physical or environmental parameter of interest such as strain, temperature, or pressure, now accessible to interferometric measurement by means of a sensing transducer accessed remotely through optical fibers. Common today are optical fiber sensors that use specialized fiber structures such as Bragg gratings. Fiber-based DMI sensors may be multiplexed through coherence or other mechanisms. This provides the opportunity for a single, perhaps highly complex source and detection system to leverage multiple sensing points cost-effectively. The remote sensors may be entirely passive, that is, without electrical power, and may have multiaxis and absolute positioning capability.

4.8.5 Optical Encoders

Linear and angular displacement can also be measured by optically detecting the lateral motion of a grid or grating pattern. This is the principle behind optical encoders, which have long been a compact and inexpensive alternative to laser DMI systems, particularly in precision engineering and the machine tool industry. As noted in Section 4.7, in recent years, they have become strong candidates for overcoming air turbulence in the most advanced and demanding stage control systems, often relying on 2D XY grids.

Homodyne and heterodyne DMI systems can serve as the basis for optical encoder sensors, as briefly described here with Figure 4.54. Using a modified Michelson interferometer, the reference and measurement beams both diffract from a grating at the Littrow angle. The system detects the lateral motion of the grating, with one complete $2\pi$ phase cycle for...
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Modern systems target sub-nanometer resolution and can have the benefit of reduced sensitivity to air turbulence. The configuration of Figure 4.54 is also sensitive to grating tilt, because of the wide separation of the points of optical contact for the grating reference and measurement beam, and is useful for monitoring this degree of freedom. Designs have been developed specifically for lateral motions only, with tolerance for tip and tilt, using the same basic double-pass principles as an HSPMI.

4.9 Summary: Using Displacement Interferometry

Throughout this survey, consistent themes characterize laser displacement interferometry as an option for precision position monitoring. Chief among this is high resolution (to <1 nm) over wide displacement ranges (>1 m typically, >1 km for geophysical and space applications), low Abbe error, and high-speed data acquisition (Sections 4.1 and 4.2). Applications that rely on these favorable characteristics include microlithography stage position control, machine tool validation, and the calibration of secondary position sensors (Section 4.7). These benefits are offset by the relative high cost of laser interferometry compared to other options, sensitivity to air turbulence, and absence of absolute distance information for the most common commercial systems.

Assuming that the application calls for a displacement interferometer, there are additional choices regarding the configuration and the necessary enabling elements to provide the desired position data. Configuration choices include homodyne or heterodyne method detection (Section 4.3), Zeeman or externally modulated light source (Section 4.4), and the geometry for the interferometer optics (Section 4.5). The choice of interferometer components and system combined with the environmental conditions and the measurement strategy will determine the level of uncertainty (Section 4.6) and dictate modifications or upgrades based on the magnitudes of the various uncertainty contributions.

The advancing requirements for applications continue to drive the advancement of new displacement interferometer systems. Developing solutions (Section 4.8) seek to overcome

FIGURE 4.54
Optical encoder for detecting lateral motions of a grating using a Michelson interferometer. (From Akiyama, K. and Iwaoka, H., High resolution digital diffraction grating scale encoder, U.S. Patent 4,629,886, 1986.)
traditional limitations as well as to push the technology to new performance targets. These developments provide opportunities for innovation as well as new applications for what is arguably the most fundamental and historical application of optical interference in precision engineering.

References

42. ZMI™ 7705 Laser Head, *Specification Sheet SS-0044* (Zygo Corporation, Middlefield, CT, 2009).
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