Birefringence in rapidly rotating glass disks

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Centripetal forces modify the optical properties of a rotating glass disk, thus creating a circularly symmetric distortion in the refractive index. This centripetal birefringence has a strong radial dependency and increases with the square of the spin speed. The effect on a polarized beam transmitted through the glass may be reduced mathematically to that of an effective wave plate whose retardance and orientation may be calculated from knowledge of the stress distribution in the disk. Alternatively, one can directly measure the Jones-matrix elements that correspond to the effective wave plate by use of polarization phase measurements at two or more locations on the disk. This direct measurement compensates the centripetal birefringence in the instrumentation employed by the data-storage industry to measure the flying height of read–write heads. © 1998 Optical Society of America [S0740-3232(98)01404-5]

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1. INTRODUCTION

A glass disk spinning at 12,000 rpm is under a significant amount of stress. The centripetal forces holding the disk together are provided by mechanical strains that are clearly visible in polarized light. Stress-induced birefringence modifies the polarization state of light passing through the glass, thus converting linear polarization to elliptical polarization or even rotating the polarization to an orthogonal state.

Centripetal birefringence is an interesting physical phenomenon that takes on practical importance for optical systems that employ rotating glass disks. Test instrumentation for the data-storage industry is a good example. An optical height measurement is a preferred method for quality testing of read–write sliders in production as well as for the development of new slider geometries that reduce flying height and increase data-storage densities.1 Flying-height testers employ a glass substrate in place of the real magnetic disk, thus providing direct interferometric measurement of the submicrometer air gap between the glass and the slider. The nanometer-level accuracy of the measurement imposed stringent requirements on the optical quality of the glass disks.

Until recently, all flying-height test equipment employed a simple intensity measurement at normal incidence.2-4 The data-storage industry has begun working with polarization-based testing to improve the quality and reliability of the measurement for near-contact read–write sliders.5-7 As Fig. 1 shows, polarized light passes through the rotating disk, reflects from the slider–glass interface, and passes back through the glass to an interferometric receiver. The receiver analyzes the intensity and the polarization state of the reflected beam, thereby providing data for calculating the narrow air gap between the glass and the slider.8-10 Because the polarized measurement beam passes through the glass disk, an accurate model of centripetal birefringence is essential.

Polarization-based flying-height testing is a new technology, and there is presently no literature on the effects of disk birefringence in this context. The most relevant prior work relates to magneto-optic (MO) data-storage systems. The injection molding process for plastic MO disks results in residual stress birefringence with a circular symmetry.11 Because the physical principle of Kerr-effect MO storage involves a small differential change in polarization, disk birefringence is an important issue.12,13 Quality control and research and development tools for MO disks employ an ellipsometric geometry similar to that shown in Fig. 1, but without the slider and with a stationary plastic disk.14,15 Although the measurement geometry is superficially the same, these techniques do not apply directly to the problem of dynamic height measurement of a slider flying over a rotating glass disk.

My objective in this paper, therefore, is to model centripetal birefringence in view of the practical problem of measuring slider flying heights with polarized light. Initially, the treatment will be quite general, involving standard equations for mechanical stress birefringence. This general treatment will make it possible to estimate the magnitude and symmetry of the optical anisotropy in the glass. I will then consider the effect of centripetal birefringence on a polarized beam passing through the disk at an arbitrary angle. I propose that the net effect of centripetal birefringence may be reduced to a two-dimensional Jones matrix that corresponds to an equivalent wave plate oriented perpendicularly to the beam. By appropriate approximations, it is possible to directly measure the two independent elements of the Jones matrix, thus providing an essential verification of the theory. Finally, I will show that this direct measurement technique is a practical means for compensation birefringence in flying-height test equipment.

2. CENTRIPETAL BIREFRINGENCE

The stress induced by rapidly rotating a disk is circularly symmetric and is both radial and tangential. Suitable orthogonal stress axes are x, y, z, where x is radial, y is
tangential, and $z$ is parallel to the rotation axis. In this coordinate system, the stress magnitudes are

$$
\sigma_x = -\rho \omega^2 \left( \frac{3 + \mu}{8} \right) \left[ q_{\text{out}}^2 + q_{\text{ins}}^2 - \frac{q_{\text{out}}^2 q_{\text{ins}}^2}{q^2} - q^2 \right],
$$

$$
\sigma_y = -\rho \omega^2 \left( \frac{3 + \mu}{8} \right) \left[ q_{\text{out}}^2 + q_{\text{ins}}^2 \right] + q_{\text{out}}^2 q_{\text{ins}}^2 \left( \frac{1 + 3\mu}{3 + \mu} \right) q^2,
$$

$$
\sigma_z = 0
$$

where $\omega$ is the spin speed in rad/s, $\rho$ is the mass density of the disk, $\mu$ is Poisson’s ratio, $q$ is the measurement radius, $q_{\text{out}}$ is the outside radius, and $q_{\text{ins}}$ is the radius of the central hole.\(^{16}\) Typical values for BK7 glass are $\mu = 0.212$ and $\rho = 2.6 \text{ gm/cm}^3$. For the fixed radii I will use $q_{\text{ins}} = 3.18 \text{ mm}$ and $q_{\text{out}} = 53 \text{ mm}$.

Because glass is an amorphous material, stress-induced changes in the refractive index are always either parallel to a stress axis or in the plane perpendicular to the stress. The connection between stress and birefringence therefore involves only the two stress-optic coefficients $\kappa_1$ and $\kappa_2$, where the subscripts 1, 2 are for parallel and perpendicular index changes, respectively. Typical values for BK7 are $\kappa_1 = -0.5$ and $\kappa_2 = -3.3 \text{ (nm/cm)/bar}$. For small index changes (e.g., $\Delta n_{x,y,z} < 10^{-4}$), the principal axes of birefringence are aligned with the stress axes and

$$
\left[ \begin{array}{c} \Delta n_x \\ \Delta n_y \\ \Delta n_z \end{array} \right] = \frac{1}{\kappa_1 \kappa_2 \kappa_3} \left[ \begin{array}{ccc} \kappa_1 & \kappa_2 & \kappa_3 \\ \kappa_2 & \kappa_1 & \kappa_3 \\ \kappa_3 & \kappa_2 & \kappa_1 \end{array} \right] \left[ \begin{array}{c} \sigma_x \\ \sigma_y \\ \sigma_z \end{array} \right].
$$

Because the relative indices $\Delta n_{x,y,z}$ are all defined with respect to the nominal 1.52 value for BK7 glass, it is reasonable and convenient to reduce the number of independent relative index values to two by use of the following transformation:

$$
\Delta n_x \to \Delta n_{\perp},
$$

$$
\Delta n_y \to -\Delta n_{\perp}
$$

$$
\Delta n_z \to \Delta n_{\perp},
$$

where

$$
2\Delta n_{\parallel} = \Delta n_x - \Delta n_y
$$

$$
2\Delta n_{\perp} = 2\Delta n_z - (\Delta n_x + \Delta n_y).
$$

Figure 2 is a graph of theoretical relative indices $\Delta n_{\perp}, \Delta n_{\parallel}$ for a range of radii $q$ at a fixed spin speed of 9000 rpm (9 krpm or $\omega/2\pi = 150 \text{ s}^{-1}$). Because the birefringence is proportional to the square of the spin speed, similar graphs may be derived from Fig. 2 by multiplying the vertical axis by the square of the ratio of the new spin speed to 9 krpm. For example, the index changes at 0.9 krpm are 100 times smaller, but the radial dependence is proportionally the same.

3. EFFECT ON A POLARIZED BEAM

Deciphering the effects of centripetal birefringence on the measurement beam shown in Fig. 1 is not an easy problem. However, because the index changes are small, a reasonable line of attack is to simplify the geometry of Fig. 1 to that of Fig. 3 by calculating the retardance and orientation of two effective waveplates acting on the measurement beam. The task of this section, therefore, is to calculate the characteristics of the effective waveplates. I have chosen to analyze propagation through a birefringent medium by use of matrices and eigenvalue analysis rather than the more common geometric construction involving the index ellipsoid. In the present approach, determining the wave-plate characteristics involves a two-angle coordinate transformation to a coordinate system defined by the measurement beam, followed by a normal-mode analysis.\(^{17}\) The end result is an equivalent retardance $\delta$ and orientation angle $\gamma$ for the wave plates B1 and B2 shown in Fig. 3.

The first step in the matrix approach is to define a square-matrix representation $\mathbf{V}$ of the centripetal birefringence in the $x$, $y$, $z$ coordinate system.\(^{18}\)
form involves the same skew angle \( z \) with respect to the plane of the disk. The measurement beam does not share the same skew angle \( z \) with respect to the plane of incidence and, consequently, \( s \) is perpendicular to the plane of incidence and \( p \) is in the plane of incidence. Figure 4 shows the two coordinate rotations that transform between the \( x, y, z \) and \( s, p, u \) systems for a beam traveling through air at an inclination angle \( \phi \) with respect to the \( z \) axis. Within the glass, the transforms involve the same skew angle \( \zeta \) but an inclination angle \( \phi_G \), given by Snell’s law, for a nominal glass index \( n_G \):

\[
\sin(\phi_G) = n_G^{-1} \sin(\phi).
\]

The rotational transform within the glass involves the following matrix operation:

\[
v' = RvR^T,
\]

where

\[
V = \begin{bmatrix}
\Delta n_x & 0 & 0 \\
0 & \Delta n_y & 0 \\
0 & 0 & \Delta n_z
\end{bmatrix}.
\]

The transformed birefringence matrix has the following symmetric form:

\[
v' = \begin{bmatrix}
\Delta n_s & \Delta n_{sp} & \Delta n_{su} \\
\Delta n_{sp} & \Delta n_p & \Delta n_{pu} \\
\Delta n_{su} & \Delta n_{pu} & \Delta n_u
\end{bmatrix}.
\]

The next step is to interpret the transformed matrix \( v' \) in terms of a wave plate. Because the beam is traveling in the \( u \) direction, only the upper-left 2 \( \times \) 2 elements of \( v' \) are important in the remaining calculations. A reduced form of the transformed matrix \( v' \) is therefore

\[
v^R = ND = ND
\]

where

\[
D = \begin{bmatrix}
D_s \\
D_p
\end{bmatrix},
\]

and the eigenvalue \( N \) is a constant. There are two eigenvalues \( N^+ \) and \( N^- \) and eigenvectors \( D^\pm \) corresponding to the relative refractive indices and principle axes of the equivalent waveplate.

Upon solving the eigenvalue problem posed by Eq. 12, we may write the index difference \( \Delta N = N^+ - N^- \) as

\[
\Delta N = \sqrt{(\Delta n_s - \Delta n_p)^2 + 4\Delta n_{sp}^2}.
\]

The accumulated phase delay or waveplate retardance between the axes after transmission through the glass is

\[
\delta = kL\Delta N,
\]

where \( L = T/|\cos(\phi)| \) is the path length through a glass disk of thickness \( T \), and \( k = 2\pi/\lambda \) is the wavenumber for the source illumination. The angle \( \gamma \) that the eigenvector \( D^+ \) makes with respect to the \( s \) axis in the \( s, p, u \) reference frame is given by

\[
\tan(\gamma) = D_p^+/D_s^+.
\]

By substitution into the normal mode equation, the angle works out to

\[
\tan(\gamma) = \frac{\Delta N + (\Delta n_s - \Delta n_p)}{2\Delta n_{sp}}.
\]

A similar calculation based on \( D^- \) gives the same angle \( \pm 90^\circ \).
For an incident angle of 50° with respect to the top surface of the glass disk, the inclination angle $\phi_G$ is approximately $-150^\circ$ before and $+150^\circ$ after reflection. There are therefore two transformed matrices $\mathbf{V'}$, one for the incident and one for the reflected portions of the measurement beam. However, for reasons of symmetry, we need only calculate the effective wave plate for one portion of the beam.\(^\text{20}\) The reflected beam will have the identical equivalent wave plate, rotated in the opposite direction.

Figure 5 shows the results of theoretical calculations for the equivalent wave-plate characteristics $\delta$, $\gamma$ as a function of radius. The graphs show a strong radial dependence, with the greatest net birefringence near the central hole in the disk. The dependence is gradual and well behaved between $9 = 10$ and 45 mm, so it is within this radial range that we should expect the best agreement between theory and reality. The retardance $\delta$ is a quadratic function of spin speed and is independent of the sign of the skew angle. The wave-plate angle $\gamma$ does not vary at all with speed but is highly sensitive to skew angle and radius.

An important conclusion of this section is that the waveplate is aligned with the $s$, $p$ axes ($\gamma = 0$ or 90°) when the skew angle $\zeta$ is equal to zero. This conclusion follows from the observation that the reduced matrix $\mathbf{V'}$ is diagonal for this special case. There is no polarization mixing for this situation, and the only effect of birefringence is the introduction of a phase delay between the $s$ and $p$ components of the polarized measurement beam. For all other skew angles the effect is a complicated function of the wave-plate characteristics and the initial polarization state. Section 4 will show that Jones calculus simplifies the mathematics for this more complicated situation.

4. JONES MATRICES

The effect of propagation through the birefringent glass has now been reduced to the action of a wave plate with a retardance $\delta$ and principal axes tilted to an angle $\gamma$ in the $s$, $p$, $u$ reference frame. The next step is to calculate the effect of the waveplate on the measurement beam. With Jones calculus,\(^\text{21}\) the resultant electric field $\mathbf{E}'$ after the beam passing through waveplate $\mathbf{B}1$ is

$$\mathbf{E}' = \mathbf{B}_1 \mathbf{E},$$

where the incident field is represented by

$$\mathbf{E} = \begin{bmatrix} \mathbf{E}_s \\ \mathbf{E}_p \end{bmatrix}.$$  \hfill (19)

The Jones matrix for the waveplate $\mathbf{B}_1$ is

$$\mathbf{B}_1(\delta, \gamma) = \mathbf{R}(\gamma) \mathbf{B}(\delta) \mathbf{R}(-\gamma),$$  \hfill (20)

where

$$\mathbf{B}(\delta) = \begin{bmatrix} \exp(-i \delta/2) & 0 \\ 0 & \exp(+i \delta/2) \end{bmatrix},$$  \hfill (21)

$$\mathbf{R}(\gamma) = \begin{bmatrix} \cos(\gamma) & \sin(\gamma) \\ -\sin(\gamma) & \cos(\gamma) \end{bmatrix}. $$  \hfill (22)

The wave plates $\mathbf{B}_1$ and $\mathbf{B}_2$ shown in Fig. 3 are identical, except that $\mathbf{B}_2$ is rotated through an angle $-\gamma$. It is convenient to rewrite the Jones matrices as

$$\mathbf{B}_1 = \begin{bmatrix} b & ia \\ ia & b^* \end{bmatrix},$$ \hfill (23)

$$\mathbf{B}_2 = \begin{bmatrix} b & -ia \\ -ia & b^* \end{bmatrix},$$ \hfill (24)

where

$$a = \sin(2\gamma)\sin(\delta/2),$$ \hfill (25)

$$b = \cos(\delta/2) + i \cos(2\gamma)\sin(\delta/2).$$ \hfill (26)

The full calculation includes a reflection from the glass surface. With the slider removed, the bare-glass reflection is represented by the following Jones matrix:

$$\mathbf{G} = \begin{bmatrix} r_s & 0 \\ 0 & r_p \end{bmatrix},$$ \hfill (27)

where the reflectivities $r_{sp}$ are the internal Fresnel reflectivities of glass in air for the $s$ and $p$ polarization states. For a 50° incident angle on BK7 glass, $r_s = -0.34$ and $r_p = 0.06$. The final electric field $\mathbf{E}_G$ for the case of a glass disk without a slider is

$$\mathbf{E}_G = (\mathbf{B}_2 \mathbf{G} \mathbf{B}_1) \mathbf{E}.$$ \hfill (28)
This calculation includes birefringence effects on both the incident and reflected portions of the measurement beam, excluding the relatively unimportant reflection losses at the top surface of the glass disk.

5. APPROXIMATE FORMULAS

In the limit of small birefringence, it is convenient to derive a series of simple approximate formulas for the polarization state and intensity of the resultant electric field. Specifically, we are looking for simple linear expressions for the intensities of the s and p components as well as the relative s–p phase angle. These formulas derived in this section will be sufficient for most practical cases of interest in flying-height test equipment. Furthermore, the formulas are the starting point for the birefringence detection algorithm described in Section 7.

The first step is to derive simple equations for the Jones-matrix elements $a$, $b$ in terms of the principal-axis indices $\Delta n_{\|}$, $\Delta n_{\perp}$ and coordinate angles $\phi_G$, $\zeta$. Combining Eqs. (25) and (26) with Eqs. (15) and (17) yields the following approximate formulas valid for $\delta/2 \ll 1$:

$$a \approx \delta_{\parallel} \sin(2\zeta) \cos(\phi_G),$$

$$b \approx \exp(i\Phi/4),$$

where

$$\Phi = 2\delta_{\parallel} \sin(\phi_G)^2 - 2\delta_{\parallel} [1 + \cos(\phi_G)^2] \cos(2\zeta),$$

$$\delta_{\parallel} = kL\eta_{\perp},$$

$$\delta_{\parallel} = kL\eta_{\parallel}. \tag{32}$$

These formulas are accurate to <1% in $a$ and $\arg(b)$ up to 12 krpm for all radii and skew angles. To this level of approximation, the magnitude of the diagonal term $b \approx 1$, the off-diagonal term $a \ll 1$, and the phase $\Phi/4$ of $b$ is also $\ll 1$.

Next I will simplify the matrix calculation Eq. (28) for the reflected field from a bare-glass disk by retaining only those terms that are first order in both $a$ and $\Phi$. I will consider specifically the practical case of a linearly polarized incident electric field $E$ with equal $s$ and $p$ components. The Jones vector for this field may be written in some arbitrary unit system simply as

$$E = \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{33}$$

Then Eq. (28) for the reflected electric field $E_G$ becomes

$$E_G = \begin{pmatrix} b & -ia \\ -ia & b^* \end{pmatrix} \begin{pmatrix} r_s & 0 \\ 0 & r_p \end{pmatrix} \begin{pmatrix} b + ia \\ b^* + ia \end{pmatrix}. \tag{34}$$

The last matrix term in the right-hand member is the Jones vector for the electric field just before reflection from the underside of the disk (see Fig. 1). The $s$–$p$ phase of this prereflection field is given by the argument of the ratio of the two elements of the vector:

$$\arg \left( \frac{b + ia}{b^* + ia} \right) \approx (1 + a)\Phi/2. \tag{35}$$

To first order in $a$ and $\Phi$, only the $\Phi/2$ term is important, and $a$ may be neglected. The $s$ and $p$ intensities of the prereflected field are not influenced to first order by either $a$ or $\Phi$:

$$|b + ia|^2 \approx 1 + a\Phi/2,$$

$$|b^* + ia|^2 \approx 1 - a\Phi/2. \tag{36}$$

We can therefore safely drop the factor $a$ from the right-hand matrix term in Eq. (34) and write

$$E_G = \begin{pmatrix} r_s b^2 & -iar_s b^* \\ -iar_pb^* & -iar_p b \end{pmatrix}. \tag{37}$$

This level of approximation is equivalent to the statement that polarization mixing is negligible for the incident portion of the beam. This is a considerable simplification of the problem.

In the remaining calculation it will be important to include the contribution of the off-diagonal term $a$. The relative s–p phase $\theta_G$ for $E_G$ is

$$\theta_G \approx -\arg \left( \frac{r_s b^2}{b^2} \frac{1 - iarb^3}{1 - iarb^3} \right), \tag{38}$$

which simplifies for small $arb^3/2$ to

$$\theta_G \approx -\pi + \Phi - \arg \left( 1 - ia \frac{r_s b^2 - r_p b^3}{r_p r_s} \right). \tag{39}$$

To further simplify this expression, note that

$$\frac{r_s b^3 + r_p b^3}{r_p r_s} = \left( \frac{r_s^2 - r_p^2}{r_p r_s} \right) + \left( \frac{r_s^2 + r_p^2}{r_p r_s} \right) \frac{3\Phi}{4}. \tag{40}$$

The last term in the right-hand member has only a second-order effect on the phase $\theta_G$ and can be neglected. The final expression is the very simple linear formula,

$$\theta_G \approx -\pi + \Phi - aU, \tag{41}$$

where

$$U = \frac{r_s^2 - r_p^2}{r_s r_p}, \tag{42}$$

and $\Phi$ and $a$ appear in relations (29) and (31). A similar analysis for the $s$ and $p$ intensities shows that to first order in $\Phi$ and $a$

$$I_s^G = r_s^2,$$

$$I_p^G = r_p^2. \tag{43}$$

The net correction to the $s$ and $p$ intensities for bare glass is negligible.

At this point the problem has essentially been solved, at least for the case of a bare-glass disk. Knowing the spin speed and the physical properties of the glass, we can now predict the intensity and polarization state of the reflected electric field $E_G$. The results show that for bare glass and a linearly polarized incident beam there is a negligible change in intensity for the two polarization components for weak birefringence. However, to the same degree of approximation, there is a significant change in $s$–$p$ phase. The birefringence-corrected phase $\theta_G$ for bare glass may be calculated from the approximate formula defined by relation (41).
So far we have been concerned primarily with the problem of calculating the effect of centripetal birefringence from our knowledge of stress patterns in a rotating glass disk. Before tackling the more complicated case of a flying-height tester, it will be worthwhile to verify the theory using direct, experimental measurements of birefringence on bare glass.

6. EXPERIMENTAL VERIFICATION

In the preceding section I showed that birefringence has a significant effect on the relative phase \( \theta_G \) of the \( s \) and \( p \) polarizations for a beam transmitted through and reflected from a bare-glass disk in rapid rotation. A convenient way to detect centripetal birefringence, therefore, is to probe the bare-glass disk with a linearly polarized beam and compare the results with the approximate formula (41). In the absence of birefringence, the reflected beam remains linearly polarized but experiences a phase shift \( -\pi \) between the \( s \) and \( p \) polarization states. A value of \( \theta_G \) different from \( -\pi \) is evidence of birefringence in the glass disk.

A direct measurement of birefringence requires a polarized light source, a receiver assembly capable of measuring the relative phase between the \( s \) and \( p \) polarization states, and a spin stand for the glass disk. The experimental data in this section were acquired on a Pegasus 2000 flying-height tester manufactured by the Zygo Corporation in Middlefield, Connecticut. Figure 6 compares the theoretical prediction for the \( s-p \) phase \( \theta_G \) with experimental data over a range of radii \( q \) from 10–45 mm. The magnitude and general trend of the experimental \( s-p \) phase are predicted by the equivalent-wave-plate model. In particular, the somewhat surprising asymmetry with respect to skew angle \( \zeta \) is present in both the theoretical and experimental data. This asymmetry is most evident in the 12 krpm spin speed, for which the \( \zeta = 20^\circ \) data is widely separated from and shows far less
radial dependence than the $-20^\circ$ data. Figure 7 provides a clear picture of the dependence on skew angle $\zeta$ for a fixed radius and spin speed. Experimental data in Fig. 8 confirm a nearly perfect quadratic dependence on spin speed, as predicted by the theory.

The small deviations from perfect agreement in Fig. 6 may be attributed in part to the implicit approximations in the theoretical geometry. For example, the actual measurement beam is a 0.05-NA cone rather than the pencil beam illustrated in Fig. 1. Also, the assumption that the skew angle $\zeta$ is the same for all portions of the measurement beam is not strictly accurate because of the position of the beam changes as it passes through the 6-mm-thick disk. A more detailed analysis does in fact show that the slight discrepancy in the skew-angle dependence of Fig. 7 is attributable entirely to the geometrical effect of disk thickness. Finally, the simple stress relations in Eq. (1) may not fully account for all the thermal and mechanical strains within the glass.

The largest deviation from theory has a rather unexpected dimension. In effect, if the disk is spun steadily at 12 krpm and then brought to a stop, the centripetal birefringence observed at high speed does not immediately disappear. There is a relaxation time of several minutes that may be related to thermal gradients in the disk. Figure 8 is an experimental illustration of the sign reversal and slow decay in birefringence that occurs after the disk is brought abruptly to a stop. For this reason, the data shown in Fig. 9 were acquired after the system was run at a steady spin speed for >20 min. The implication of this relaxation effect is that it is not significantly accurate to predict birefringence by using the spin speed and the disk geometry alone.

7. BIREFRINGENCE DETECTION ALGORITHM

If a precise knowledge of disk birefringence is essential for subsequent measurements, then the best approach is to measure it directly. The purpose of this section is to develop an in situ birefringence detection algorithm by using only the assumption of circular symmetry. The algorithm returns the characteristics of the equivalent waveplates shown in Fig. 3 in the form of the elements $a$, $b$ of the $B_{1,2}$ matrices introduced in Section 4.

The detection procedure involves a linearly polarized measurement beam having equal $s$ and $p$ components, just as was done for the experimental work in Section 6. By inverting the simple approximation formulas in Section 5, I will calculate directly the elements $a$, $b$ by comparing the input and output polarization states of the beam.

There are two real variables $a$ and $\Phi$ in approximate formula (41) for the $s$–$p$ phase $\theta_G$. We therefore require a minimum of two measurements to solve for these variables. One tactic is to measure the phase at complementary skew angles on either side of $\zeta = 0$. If we define $\theta_G = \theta^{\pm}$ for complimentary skew angles $\pm \zeta$, the Jones matrix elements are given by

$$a \approx \frac{1}{2}[\theta^-(\phi) - \theta^+(\phi)]/U,$$

$$\Phi \approx \frac{1}{2}[\theta^-(\phi) + \theta^+(\phi)] + \pi.$$  \hspace{1cm} (44)

These formulas exploit the symmetry properties of $a$, $b$ with skew position. It is important to note that the constant $U$ takes on defined, nonzero values only for nonnormal incident angles other than Brewster’s angle.

A fairly complicated physical problem has been reduced to two simple formulas for in situ detection and quantification of centripetal birefringence. The technique involves phase measurements at complementary skew positions on bare glass. This is of great practical interest for polarization-based flying-height testing, as will be evident in the following section.

8. APPLICATION TO FLYING-HEIGHT TESTING

All of the calculations so far have been for bare glass only, without the slider shown in Fig. 1. In flying-height measurements the slider is placed nearly in contact with the rotating disk, and the measurement beam is reflected under the combined influence of the glass and the slider surface. The simple bare-glass Jones-matrix $G$ of Eq. (27) is replaced by a slider–glass matrix $S$ defined by

$$S = \begin{bmatrix} z_s & 0 \\ 0 & z_p \end{bmatrix},$$  \hspace{1cm} (46)

and $z_{sp}$ are the effective reflectivities of the slider–glass interface. The effective reflectivities $z_{sp}$ are the thin-film equations for layered media for the two polarization components $s$ and $p$:

$$z_p(\beta) = \frac{r_p + r_p' \exp(i\beta)}{1 + r_p r_p' \exp(i\beta)},$$  \hspace{1cm} (47)

$$z_s(\beta) = \frac{r_s + r_s' \exp(i\beta)}{1 + r_s r_s' \exp(i\beta)},$$  \hspace{1cm} (48)

where the phase term $\beta$ is given by

$$\beta = 2kh \cos(\phi).$$  \hspace{1cm} (49)
The amplitude reflectivities $r_{sp}$ are for the glass–air boundary, while the primed values $r'_{sp}$ refer to the air–slider boundary. Reflectivities for a slider index $n' = 2.2 + 0.4i$ are $r'_{sp} = 0.21 + 0.075i$ and $r'_{sp} = -0.54 - 0.073i$. The phase $\beta$ depends on the wave number $k = 1/A$ and the flying height $h$. The height dependence of the effective reflectivities $z_{sp}$ is the foundation of polarization-based flying-height testing.\(^5\)

Apart from replacing the bare-glass matrix $G$ with the slider–glass matrix $S$, the Jones-matrix calculation for the perturbation of the measurement beam polarization is very similar to the simple bare-glass case:

$$E_S = (B_2 SB_1)E.$$  \hspace{1cm} (50)

In the absence of birefringence, the matrices $B_{1,2}$ are identity matrices. This is the limit case appearing in previous publications on the technique.\(^5\) For this limit case, and assuming once again that the incident beam is linearly polarized with equal $s$ and $p$ components, we can define a relative $s–p$ phase

$$\theta = \arg(z_s/z_p)$$ \hspace{1cm} (51)

and $s$ and $p$ intensities

$$I_{sp} = |z_{sp}|^2.$$ \hspace{1cm} (52)

Following a procedure much like the one taken for bare glass (see Section 5), we can show that approximate calculations of the measured $s–p$ phase and intensities are

$$\theta^b = \theta + \Delta \theta,$$
$$I_{s}^b = I_s - \Delta I,$$
$$I_{p}^b = I_p + \Delta I,$$ \hspace{1cm} (53-54)

where

$$\Delta \theta = \Phi + a \frac{I_s - I_p}{IJ_p} \cos(\theta)$$ \hspace{1cm} (55)

and where $\theta$, $I_s$, $I_p$ are corresponding parameters for zero birefringence. Numerical simulations show that the approximation for $\theta$ is nearly perfect for spin speeds up to 12 krpm at zero skew and is accurate to $\pm 1^\circ$ for a $\pm 15^\circ$ skew range at spin speeds up to 10 krpm and with radii between 10 and 40 mm. The $I_s, I_p$ approximations are less accurate but are useful for estimating the magnitude of the birefringence effect on intensity.

Figure 10 shows the evolution of the phase angles $\theta$ and $\theta^b$ over a 0.5-μm range of flying heights. The largest ef-
fect on the phase is caused by the constant offset term $\Phi$, with a height-dependent distortion related to the off-diagonal term $a$ in the $B_{1,2}$ matrices. Figure 11 compares the $s/p$ intensity ratio with the sum of the $s$ and $p$ intensities as a function of flying height. The variation in the intensity ratio $I_s/I_p$, which is related to the tan($\Psi$) parameter in conventional ellipsometry, is dominated by disk birefringence. These plots used the full theory rather than the approximations of relation (54). The total intensity

$$I = I_s + I_p$$

is nearly independent of birefringence phenomena, thus making it a more attractive measurement parameter for flying-height measurement than the $s/p$ ratio.

The flying-height calculation for sliders consists of comparing measured phase and intensity values with theoretical numbers and adjusting the height $h$ to find the best match between experiment and theory. The calculation involves minimizing a merit function of the form

$$\chi^2(\beta) = [I_{\text{exp}} - I]^2 + [\theta_{\text{exp}} - \theta^\beta]^2,$$  \hspace{1cm} (57)

where $I_{\text{exp}}$, $\theta_{\text{exp}}$ are the experimental or measured values corresponding to the parameters $I$, $\theta^\beta$. In more traditional flying-height test systems there is no phase information, and the minimization involves only intensity data for three or more source wavelengths. An advantage of including the phase information is that the light-source requirements are greatly simplified while the sensitivity of the measurement at low heights ($\leq 25$ nm) is improved. However, as is evident from Fig. 10, the theoretical model must include the phase changes introduced by centripetal birefringence.

There are essentially two approaches to handling centripetal birefringence in flying-height testing. The first is to predict the effect by use of the stress analysis together with knowledge of the spin speed, radius, and skew angle. The second is more empirical, involving an \textit{in situ} evaluation of the Jones-matrix coefficients $a$, $b$, or their equivalents, as described in Section 6. The existence of a relaxation time for birefringence makes the second choice more attractive. For the simplest case of zero skew, birefringence compensation reduces to a measurement of the $s-p$ phase for bare glass just before positioning the slider on the disk. The difference between the bare-glass phase $\theta_0$ and $-\pi$ is a constant offset that must be added to the theoretical value $\theta$ to make it consistent with the experimental situation.

Figure 12 compares a series of experimental height measurements for the case where the effect of birefringence has been either included or neglected. For this experiment, a read–write slider is flown at zero skew over a range of spin speeds from 2 to 12 krpm, and height measurements are performed at the slider midpoint. It is expected that the flying height above the disk will vary in an approximately linear fashion ($\sim 1\%$) as the spin speed is increased. The data show that when centripetal birefringence is properly included in the theory together with the direct detection algorithm, the observed linearity is $\pm 0.9$ nm over the 200-nm height range. If the effects of centripetal birefringence are neglected, the processed data are highly nonlinear above 6 krpm. This experiment underscores the importance of birefringence compensation in flying-height testing as well as the effectiveness of the \textit{in situ} detection technique.

It is worth noting that the potential measurement error attributable to birefringence depends on the flying height. Polarization interferometry is typically four times less sensitive to birefringence at contact (zero flying height) than at 250 nm. The approximation formulas for intensity and phase are also most accurate at contact, where the accuracy requirements are most stringent.

It is also interesting to note that disk birefringence plays a role in the design of traditional flying-height testers that do not use phase information at all. The problem arises when one uses polarized light to control unwanted reflections in the optical system of a normal-incidence, intensity-based tester. The centripetal birefringence couples into changes in intensity when the reflected light is analyzed to suppress the unwanted reflections. The resulting flying-height errors are of the same order of magnitude as those plotted in Fig. 12 (e.g., 10 nm at 10 krpm). In this case, the problem may be solved in hardware by depolarizing the incident light.

9. CONCLUSION

I began this paper with a calculation of birefringence by using well-known stress equations for a rotating glass disk. This first-principles analysis does provide a valuable confirmation of the theory in the form of the comparison plots of Figs. 6–8. In the end, what has been retained for practical use is the presumption of circular symmetry and the concept of the equivalent optical system shown in Fig. 2. The \textit{in situ} measurement technique of Section 7 is an empirical approach to birefringence compensation that accounts for the differences between the first-principles theory and practical reality. This
technique is central to the success of polarization-based flying-height testing of read–write sliders.

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REFERENCES AND NOTES

17. For an analysis based on the index ellipsoid, see, for example, M. Born and E. Wolf, Principles of Optics, 6th ed. (Pergamon, New York, 1980), p. 674.
18. The matrix \( \mathbf{v} \) is the differential portion of the electric impermeability tensor in the principal axis frame. Because the index changes are small (\(<10\%\)), this differential form is a sufficient description of the optical anisotropy of the glass.
20. The symmetry argument that allows us to equate waveplates B1 and B2 in Fig. 3 requires that we neglect the slight difference in position within the glass of the incident and the reflected beams. This approximation simplifies the model but introduces an error that may be significant at small radii.
22. Estimates of the accuracy of approximations in this paper all assume the disk geometry defined in Section 2, i.e., inner radius \( q_{in} = 3.18 \) mm, outer radius \( q_{out} = 53 \) mm, disk thickness \( T = 7 \) mm.
23. As an alternative to linear input polarization, Lacey and Womack have proposed using circular polarization (U.S. Patent 5,636,178, “Imaging polarimeter detector for measurement of small spacings” [June 10, 1997]). However, a linear input polarization substantially reduces the effect of polarization mixing attributable to disk birefringence.
26. Centripetal birefringence measurement and compensation are covered by U.S. Patent 5,644,562 to P. de Groot entitled “Method and apparatus for measuring and compensating birefringence in rotating disks” (July 1, 1997).