Determination of the phase change on reflection from two-beam interference

J. F. Biegen

Zygo Corporation, Laurel Brook Road, Middlefield, Connecticut 06455

Received June 15, 1994

Light reflected by a nondielectric material experiences a phase change on reflection that differs from light reflected by a dielectric material and other nondielectric materials. The complex degree of coherence for small optical path differences is derived for two-beam interference when the illumination source is extended, incoherent, and quasi-monochromatic. An analysis of the two-beam interference pattern reveals a simple relationship between the phase of the interference pattern at the point of maximum fringe visibility and the material-dependent phase change on reflection.

Recently developed methods in interference microscopy, collectively called scanning white-light interferometry, result in a greatly expanded vertical measurement range over that of previous optical profiling techniques generally known as phase-measuring interference microscopy. The scanning white-light interferometry method involves digitizing the interference pattern along the optical axis at all points in the field of view, analyzing the digitized interference pattern to determine the position in the scan where the fringe visibility is a maximum, and constructing a surface profile from the axial positions of maximum fringe visibility. Measurement errors result when scanning white-light interferometry or phase-measuring interference microscopy is used to profile surfaces composed of dissimilar materials. The position of maximum fringe visibility and the phase of the interference pattern relative to a fixed axial point will shift in position as a function of material, thus producing inaccuracies in the surface profile.

However, as I will show in this Letter, it is possible to determine the phase change on reflection from a scanning white-light interferometry measurement. The phase of the interference pattern at the point of maximum fringe visibility is proportional to the material-dependent phase change on reflection. This surprising result is achieved with a microscope interferometer having a quasi-monochromatic extended illumination source and a moderate-to-large numerical aperture. Presented below is a method to determine the phase change on reflection by an analysis of the interference pattern.

The equation for two-beam interference with partially coherent illumination is

\[ I(\Delta z) = I_1 + I_2 + 2\sqrt{I_1 I_2} \text{Re}[\gamma_{12}(\Delta z)], \]  

where \( I_1 \) is the test beam intensity at a given test point \( P_1 \), \( I_2 \) is the reference beam intensity at a given reference point \( P_2 \), and \( \Delta z \) is the on-axis geometrical path difference between the test point \( P_1 \) and reference point \( P_2 \). The term \( \text{Re}[\gamma_{12}(\Delta z)] \) refers to the real part of the expression contained within the brackets, and \( \gamma_{12}(\Delta z) \) is the complex degree of coherence whose modulus \( |\gamma_{12}(\Delta z)| \) satisfies the relation \( 0 \leq |\gamma_{12}(\Delta z)| \leq 1 \). The fringe visibility \( V(\Delta z) \) of the interference pattern is proportional to the modulus of the complex degree of coherence, that is, \( V(\Delta z) \propto |\gamma_{12}(\Delta z)| \).

For simplicity, consider a microscope interferometer without lateral or radial shear between the images of test point \( P_1 \) and reference point \( P_2 \). This confines the analysis to the depth of focus of the microscope interferometer objective and the interference pattern produced by small incremental changes in the axial separation between test point \( P_1 \) and reference point \( P_2 \) around the axial point at which zero optical path difference occurs. Referring to Fig. 1, \( R_1 \) denotes the distance between a point \( Q(r, z) \) on a spherical monochromatic wave \( W \) emerging from a circular diffraction aperture of radius \( a \) and converging to test point \( P_1(z) \). \( R_2 \) denotes the distance between the same point \( Q(r, z) \) on the same spherical wave \( W \) converging to the reference point \( P_2(z) \). The complex degree of coherence for an extended quasi-monochromatic incoherent source in a double-pass interferometer is given by

\[ \gamma_{12}(\Delta z) \propto \int_{\text{w}} \frac{I(Q)}{R_1 R_2} \exp \left[ i \frac{4\pi}{\lambda_0} (\Delta R + \Delta \phi) \right] dQ, \]  

(2)

where \( I(Q) \) represents the source intensity distribution across the aperture \( 2a \), the geometrical path difference is \( \Delta R = R_1 - R_2 \), \( \lambda_0 \) is the mean wavelength of the quasi-monochromatic source, and \( \Delta \phi = \phi_1 - \phi_2 \), where \( \phi_1 \) and \( \phi_2 \) represent any phase change that occurs at a point \( Q(r, z) \) on the spherical wavefront \( W \) converging to and reflecting from points \( P_1 \) and \( P_2 \) independent of the geometrical path difference \( \Delta R \).

Referring again to Fig. 1, \( R \) is the distance between \( C \) and points \( P_1 \) and \( P_2 \), so that \( R_1^2 = r^2 + [(R + \Delta z) - z]^2 \). The distance \( R \) is large compared with \( \Delta z \), \( 2a \) is small compared with \( R \), and \( \alpha \gg \lambda \). Expanding and neglecting the terms above second power
The amplitude and phase components for a test beam traversing the beam path $Q_PQ'$ is
\[
\Psi_1(n, k, \theta) = [\tau_{bs}^2 \rho_{ts} \exp i(2\Phi_{Tbs} + \theta_{ts})]_{TE} + [\tau_{bs}^2 \rho_{ts} \exp i(2\Phi_{Tbs} + \theta_{ts})]_{TM},
\]
where $\rho$ and $\tau$ are the reflection and transmission amplitudes and $\phi$ is the phase. A similar analysis is done for the reference beam. The subscripts $ts$ and $bs$ represent test surface and beam splitter, respectively, and $T$ refers to transmission through the beam splitter. The resultant test-cavity phase change is found by use of the following relation:
\[
\phi_1(n, k, \theta) = \arctan(\text{Im}[\Psi_1(n, k, \theta)]/\text{Re}[\Psi_1(n, k, \theta)]),
\]
where Re and Im stand for the real and imaginary parts of the expressions contained within the brackets. Using Eq. (6) to find explicit expressions for $\phi_1(n, k, \theta)$ and $\phi_2(n, k, \theta)$, substituting these expressions into relation (4), and doing the necessary integration would be difficult at best. However, Eq. (6) over the anticipated NA range can be adequately replaced by a parabolic approximation, yielding an equation of the form
\[
\phi(r) = \phi_0(1 + \kappa N A^2 r^2),
\]
where $\phi_0$ is the interferometer-cavity phase change at normal incidence for the mean wavelength $\lambda_0$ and $\kappa$ is a constant whose value depends on the NA of the interferometer objective being used and on the optical properties of the materials that compose the interferometer. Substituting $\phi_{ts}(1 + \kappa_1 N A^2 r^2)$ and $\phi_{bs}(1 + \kappa_2 N A^2 r^2)$ for $\phi_1(r)$ and $\phi_2(r)$ in relation (4), integrating, and taking the real part yields the following expression:
\[
\text{Re}[\gamma_{12}(\Delta z)] = \cos \left[ \frac{4\pi}{\lambda_0} \left( 1 - \frac{N A^2}{4} \right) \Delta z \right] + \left[ \frac{\Delta \phi_0 + (\kappa_1 \phi_{ts} - \kappa_2 \phi_{bs})}{2} N A^2 \right] \times \text{sinc} \left[ \frac{\pi}{\lambda_0} \Delta z - \left( \frac{\kappa_1 \phi_{ts} - \kappa_2 \phi_{bs}}{2} \right) N A^2 \right].
\]
Equation (8) is the real part of the complex degree of coherence for two-beam interference, assuming a circular, uniform, quasi-monochromatic illumination source and an angle-of-incidence-dependent phase change on reflection. The term $\Delta \phi_0$ is the phase difference between the test-cavity phase change at normal incidence, $\phi_{ts}$, and the reference-cavity phase change at normal incidence, $\phi_{bs}$, such that $\Delta \phi_0 = \phi_{ts} - \phi_{bs}$. When $\Delta z = (\kappa_1 \phi_{ts} - \kappa_2 \phi_{bs})\lambda_0/2\pi$ in the argument of the sinc function in Eq. (8), the fringe visibility of the interference pattern will be a maximum. Substituting Eq. (8) into Eq. (1) by use of the above value for $\Delta z$ yields the following result:
\[
I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos[\Delta \phi_0 + 2(\kappa_1 \phi_{ts} - \kappa_2 \phi_{bs})].
\]
Equation (9) reveals that the phase of the interference pattern at the point of maximum fringe visibility is proportional to the difference in the phase change between the test cavity and the reference cavity at normal incidence.

Let $\Delta \phi_0 + 2(\kappa_1 \phi_{01} - \kappa_2 \phi_{02}) = \Phi_1$ for a nondielectric test surface. The test-cavity phase change at normal incidence for a given microscope interferometer objective is $\phi_{01} = 2\phi_{0Tba} + \phi_{0ts}$, where the term $\phi_{0Tba}$ is the test-cavity phase change in transmission at normal incidence and $\phi_{0ts}$ is the test-surface phase change on reflection at normal incidence. For a dielectric test surface, $\phi_{0ts} = 0$, the cosine argument in Eq. (9) becomes $\Phi_{\text{dielectric}} = 2\phi_{0Tba} - \phi_{02} + 2(2\kappa_1 \phi_{0Tba} - \kappa_2 \phi_{02})$. The value for $\kappa_1$ is assumed to be effectively constant over the expected range of test-surface phase-change-on-reflection values regardless of whether the test surface is dielectric or nondielectric. Subtracting $\Phi_1$ from $\Phi_{\text{dielectric}}$ to eliminate the contribution of the reference-cavity phase change and solving for $\phi_{0ts}$ yields

$$\phi_{0ts} = \frac{\Phi_1 - \Phi_{\text{dielectric}}}{1 + 2\kappa_1}.$$  (10)

At the point of maximum fringe visibility in an interference pattern, the difference between the phase for an unknown test-surface material and the phase for a dielectric test-surface material divided by the constant $1 + 2\kappa_1$ is equal to the phase change on reflection for the unknown material at zero angle of incidence.

The most straightforward way to solve for $\Phi$ is to sample and digitize the interference pattern at a known interval and to do a least-squares fit of Eq. (9) to the data. The value for $\kappa_1$ can be derived analytically if the optical properties of the microscope interferometer are well known, or it can be determined empirically if the phase change on reflection for a given nondielectric test-surface material is accurately known. To determine $\kappa_1$ empirically, we make a one-time measurement with a given interferometer objective on a test surface for which $\phi_{0ts}$ is known accurately and $\Phi_1$ has been obtained. A second measurement is made on a dielectric test surface to obtain $\Phi_{\text{dielectric}}$, and by use of Eq. (10) the value for $1 + 2\kappa_1$ is found. The value found for $1 + 2\kappa_1$ is then used in any subsequent measurements on any test surface whose optical properties are unknown.

The apparatus in Fig. 2 was used to acquire interference pattern intensity data over an array of field points, which I then analyzed to calculate the phase-change-on-reflection values found in Table 1. The interference objective used was a 40X Mirau with a NA equal to 0.5. I empirically determined the value for $1 + 2\kappa_1$ to be 1.44, using the average handbook value for gold (0.617 rad) and dividing that value into the measured phase found for gold at the point of maximum fringe visibility in the interference pattern. The interference pattern was sampled at 28 frames per fringe. Note that the phase change on reflection for a material will generally increase as a function of oxide layer growth.

Summarizing, a microscope interferometer with an extended quasi-monochromatic illumination source can be used to extract the phase change on reflection for any given material by an analysis of the two-beam interference pattern sampled along the optical axis.

The author gratefully acknowledges the assistance received from Gary Sommargren and Peter deGroot, and from Les Deck, who helped with the experimental setup and data taking.

References