Determination of the phase change on reflection from two-beam interference

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Light reflected by a nondielectric material experiences a phase change on reflection that differs from light reflected by a dielectric material and other nondielectric materials. The complex degree of coherence for small optical path differences is derived for two-beam interference when the illumination source is extended, incoherent, and quasi-monochromatic. An analysis of the two-beam interference pattern reveals a simple relationship between the phase of the interference pattern at the point of maximum fringe visibility and the material-dependent phase change on reflection.

Recently developed methods in interference microscopy, collectively called scanning white-light interferometry,[1] result in a greatly expanded vertical measurement range over that of previous optical profiling techniques generally known as phase-measuring interference microscopy.[2] The scanning white-light interferometry method involves digitizing the interference pattern along the optical axis at all points in the field of view, analyzing the digitized interference pattern to determine the position in the scan where the fringe visibility is a maximum, and constructing a surface profile from the axial positions of maximum fringe visibility. Measurement errors result when scanning white-light interferometry or phase-measuring interference microscopy is used to profile surfaces composed of dissimilar materials.[3,4] The position of maximum fringe visibility and the phase of the interference pattern relative to a fixed axial point will shift in position as a function of material, thus producing inaccuracies in the surface profile.

However, as I will show in this Letter, it is possible to determine the phase change on reflection from a scanning white-light interferometry measurement. The phase of the interference pattern at the point of maximum fringe visibility is proportional to the material-dependent phase change on reflection. This surprising result is achieved with a microscope interferometer having a quasi-monochromatic extended illumination source and a moderate-to-large numerical aperture. Presented below is a method to determine the phase change on reflection by an analysis of the interference pattern.

The equation for two-beam interference with partially coherent illumination is

$$I(\Delta z) = I_1 + I_2 + 2\sqrt{I_1 I_2}\ \mathrm{Re}[\gamma_{12}(\Delta z)], \qquad (1)$$

where I_1 is the test beam intensity at a given test point P_1, I_2 is the reference beam intensity at a given reference point P_2, and Δz is the on-axis geometrical path difference between the test point P_1 and reference point P_2. The term $\mathrm{Re}[\gamma_{12}(\Delta z)]$ refers to the real part of the expression contained within the brackets, and $\gamma_{12}(\Delta z)$ is the complex degree of coherence whose modulus $|\gamma_{12}(\Delta z)|$ satisfies the relation $0 \le |\gamma_{12}(\Delta z)| \le 1$. The fringe visibility $V(\Delta z)$ of the interference pattern is proportional to the modulus of the complex degree of coherence, that is, $V(\Delta z) \propto |\gamma_{12}(\Delta z)|$.

For simplicity, consider a microscope interferometer without lateral or radial shear between the images of test point P_1 and reference point P_2. This confines the analysis to the depth of focus of the microscope interferometer objective and the interference pattern produced by small incremental changes in the axial separation between test point P_1 and reference point P_2 around the axial point at which zero optical path difference occurs. Referring to Fig. 1, R_1 denotes the distance between a point $Q(r, z)$ on a spherical monochromatic wave W emerging from a circular diffraction aperture of radius a and converging to test point $P_1(z)$. R_2 denotes the distance between the same point $Q(r, z)$ on the same spherical wave W converging to the reference point $P_2(z)$. The complex degree of coherence for an extended quasi-monochromatic incoherent source in a double-pass interferometer is given by

$$\gamma_{12}(\Delta z) \propto \iint_W \frac{I(Q)}{R_1 R_2} \exp\left[i\left(\frac{4\pi}{\lambda_0} \Delta R + \Delta\phi \right) \right] \mathrm{d}Q\,, \qquad (2)$$

where $I(Q)$ represents the source intensity distribution across the aperture $2a$, the geometrical path difference is $\Delta R = R_1 - R_2$, λ_0 is the mean wavelength of the quasi-monochromatic source, and $\Delta\phi = \phi_1 - \phi_2$, where ϕ_1 and ϕ_2 represent any phase change that occurs at a point $Q(r, z)$ on the spherical wave front W converging to and reflecting from points P_1 and P_2 independent of the geometrical path difference ΔR.[5]

Referring again to Fig. 1, R is the distance between C and points P_1 and P_2, so that $R_1{}^2 = r^2 + [(R + \Delta z) - z]^2$. The distance R is large compared with Δz, $2a$ is small compared with R, and $a \gg \lambda$. Expanding and neglecting the terms above second power

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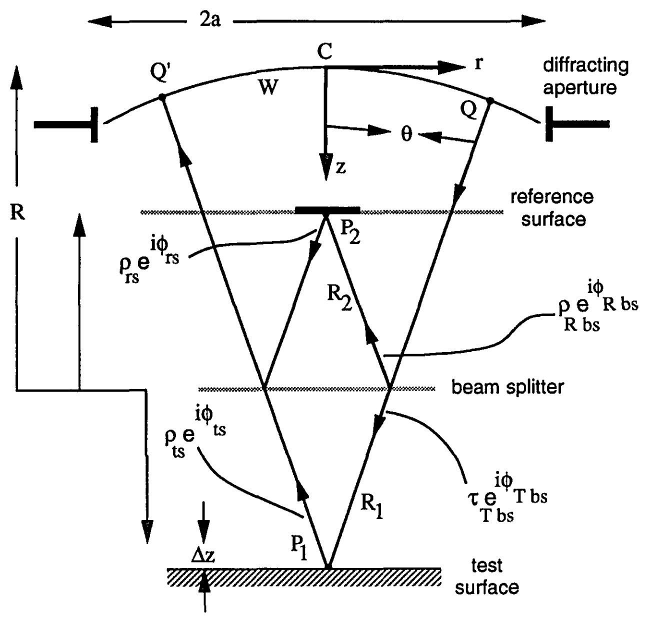

Fig. 1. Diffraction of a spherical converging wave by a circular aperture in a two-beam Mirau interferometer showing the amplitude and phase terms of a ray traversing the interferometer.

in r and first power in z so that $R_1 \cong R + \Delta z + [\Delta z^2 - 2(R + \Delta z)z + r^2 + z^2]/2R$, making the approximation $z \approx r^2/2R$, doing the same for R_2, and subtracting R_2 from R_1 results in

$$\Delta R \approx \Delta z - \Delta z \frac{r^2}{2R^2} \cdot \qquad (3)$$

Let the source intensity distribution $I(r)$ be uniform across the diffraction aperture of radius a such that $I(r) = 1$ for r less than or equal to a and $I(r) = 0$ for r greater than a. The distances R_1 and R_2 can be replaced by R in the denominator of the integrands in relation (2) since $R_1 \approx R_2$. Taking into account the functional dependence of the phase-change terms ϕ_1 and ϕ_2 on the aperture radius r together with the above assumptions, we see that relation (2) becomes

$$\gamma_{12}(\Delta z) \propto \exp\left[i\left(\frac{4\pi}{\lambda_0} \Delta z \right) \right] \times \int_0^a \int_0^{2\pi} \exp\left\{ -i\left[\frac{2\pi}{\lambda_0} \frac{\Delta z}{R^2} r^2 - \Delta\phi(r) \right] \right\} r dr d\varphi , \qquad (4)$$

where $\Delta\phi(r) = \phi_1(r) - \phi_2(r)$.

The phase change on reflection or in transmission for a nondielectric material is a function of the complex index of refraction $\hat{n} = n + ik$ and the illumination angle of incidence θ. Using the small-angle approximation, let $\theta = r/R$ and $\mathrm{NA} = a/R$ define the angle of incidence and the numerical aperture of the microscope interferometer objective, respectively. The amplitude and phase change on reflection and in transmission for a monochromatic converging wave incident upon an absorbing medium for both the TE and TM cases are coherent with respect to each other, and the total phase change on reflection is the vector sum of both components.[6] The vector sum of the amplitude and phase components for a test beam traversing the beam path QP_1Q' is

$$\Psi_1(n, k, \theta) = [\tau_{\mathrm{bs}}{}^2 \rho_{\mathrm{ts}} \exp i(2\phi_{T\mathrm{bs}} + \phi_{\mathrm{ts}})]_{\mathrm{TE}} + [\tau_{\mathrm{bs}}{}^2 \rho_{\mathrm{ts}} \exp i(2\phi_{T\mathrm{bs}} + \phi_{\mathrm{ts}})]_{\mathrm{TM}} , \qquad (5)$$

where ρ and τ are the reflection and transmission amplitudes and ϕ is the phase. A similar analysis is done for the reference beam. The subscripts ts and bs represent test surface and beam splitter, respectively, and T refers to transmission through the beam splitter. The resultant test-cavity phase change is found by use of the following relation:

$$\phi_1(n, k, \theta) = \arctan\langle \mathrm{Im}[\Psi_1(n, k, \theta)]/\mathrm{Re}[\Psi_1(n, k, \theta)] \rangle , \qquad (6)$$

where Re and Im stand for the real and imaginary parts of the expressions contained within the brackets. Using Eq. (6) to find explicit expressions for $\phi_1(n, k, \theta)$ and $\phi_2(n, k, \theta)$, substituting these expressions into relation (4), and doing the necessary integration would be difficult at best. However, Eq. (6) over the anticipated NA range can be adequately replaced by a parabolic approximation, yielding an equation of the form

$$\phi(r) = \phi_0(1 + \kappa\, \mathrm{NA}^2 r^2) , \qquad (7)$$

where ϕ_0 is the interferometer-cavity phase change at normal incidence for the mean wavelength λ_0 and κ is a constant whose value depends on the NA of the interferometer objective being used and on the optical properties of the materials that compose the interferometer. Substituting $\phi_{01}(1 + \kappa_1 \mathrm{NA}^2 r^2)$ and $\phi_{02}(1 + \kappa_2 \mathrm{NA}^2 r^2)$ for $\phi_1(r)$ and $\phi_2(r)$ in relation (4), integrating, and taking the real part yields the following expression:

$$\mathrm{Re}[\gamma_{12}(\Delta z)] = \cos\left\{ \frac{4\pi}{\lambda_0}\left(1 - \frac{\mathrm{NA}^2}{4} \right) \Delta z + \left[\Delta\phi_0 + \frac{(\kappa_1\phi_{01} - \kappa_2\phi_{02})}{2} \mathrm{NA}^2 \right] \right\} \times \mathrm{sinc}\left\{ \left[\frac{\pi}{\lambda_0} \Delta z - \frac{(\kappa_1\phi_{01} - \kappa_2\phi_{02})}{2} \right] \mathrm{NA}^2 \right\} \cdot \qquad (8)$$

Equation (8) is the real part of the complex degree of coherence for two-beam interference, assuming a circular, uniform, quasi-monochromatic illumination source and an angle-of-incidence-dependent phase change on reflection. The term $\Delta\phi_0$ is the phase difference between the test-cavity phase change at normal incidence, ϕ_{01}, and the reference-cavity phase change at normal incidence, ϕ_{02}, such that $\Delta\phi_0 = \phi_{01} - \phi_{02}$.

When $\Delta z = (\kappa_1\phi_{01} - \kappa_2\phi_{02})\lambda_0/2\pi$ in the argument of the sinc function in Eq. (8), the fringe visibility of the interference pattern will be a maximum. Substituting Eq. (8) into Eq. (1) by use of the above value for Δz yields the following result:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos[\Delta\phi_0 + 2(\kappa_1\phi_{01} - \kappa_2\phi_{02})] . \qquad (9)$$

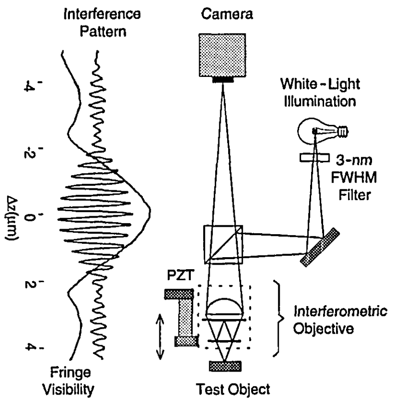

Fig. 2. Optomechanical layout of the apparatus used (right) and an interference pattern obtained with the apparatus, together with its fringe visibility curve (left). PZT, piezoelectric transducer.

Table 1. Comparison of Measured and Calculated Phase-Change Values

Material	Measured (rad)	Calculated (rad)[a]	Oxide Formation
Au	–	0.602–0.631	No
Pt	0.410	0.347–0.366	No
Cu	0.644	0.566–0.575	Yes
Al	0.282	0.253–0.279	Yes
Ni	0.503	0.418–0.433	Yes
Cr	0.341	0.339–0.366[b]	Yes
FeNi	0.546	0.383, 0.525[c]	Yes
SiC	0.043	0.012[d]	No

[a]Calculated from handbook values of n and k.[7,8]
[b]Assumes a 3-nm Cr_2O_3 oxide overlayer; Cr oxidizes readily, but further oxidation ceases after an overlayer is formed.
[c]Pole tip recession of thin-film magnetic heads measured before and after flash coating with Al to determine the phase change on reflection for the sample heads used in the experiment.
[d]Measured with an ellipsometer.

Equation (9) reveals that the phase of the interference pattern at the point of maximum fringe visibility is proportional to the difference in the phase change between the test cavity and the reference cavity at normal incidence.

Let $\Delta\phi_0 + 2(\kappa_1\phi_{01} - \kappa_2\phi_{02}) = \Phi_1$ for a nondielectric test surface. The test-cavity phase change at normal incidence for a given microscope interferometer objective is $\phi_{01} = 2\phi_{0T\text{bs}} + \phi_{0\text{ts}}$, where the term $\phi_{0T\text{bs}}$ is the test-cavity phase change in transmission at normal incidence and $\phi_{0\text{ts}}$ is the test-surface phase change on reflection at normal incidence. For a dielectric test surface, $\phi_{0\text{ts}} = 0$, the cosine argument in Eq. (9) becomes $\Phi_{\text{dielectric}} = 2\phi_{0T\text{bs}} - \phi_{02} + 2(2\kappa_1\phi_{0T\text{bs}} - \kappa_2\phi_{02})$. The value for κ_1 is assumed to be effectively constant over the expected range of test-surface phase-change-on-reflection values regardless of whether the test surface is dielectric or nondielectric. Subtracting Φ_1 from $\Phi_{\text{dielectric}}$ to eliminate the contribution of the reference-cavity phase change and solving for $\phi_{0\text{ts}}$ yields

$$\phi_{0\text{ts}} = \frac{\Phi_1 - \Phi_{\text{dielectric}}}{1 + 2\kappa_1}. \qquad (10)$$

At the point of maximum fringe visibility in an interference pattern, the difference between the phase for an unknown test-surface material and the phase for a dielectric test-surface material divided by the constant $1 + 2\kappa_1$ is equal to the phase change on reflection for the unknown material at zero angle of incidence.

The most straightforward way to solve for Φ is to sample and digitize the interference pattern at a known interval and to do a least-squares fit of Eq. (9) to the data. The value for κ_1 can be derived analytically if the optical properties of the microscope interferometer are well known, or it can be determined empirically if the phase change on reflection for a given nondielectric test-surface material is accurately known. To determine κ_1 empirically, we make a one-time measurement with a given interferometer objective on a test surface for which $\phi_{0\text{ts}}$ is known accurately and Φ_1 has been obtained. A second measurement is made on a dielectric test surface to obtain $\Phi_{\text{dielectric}}$, and by use of Eq. (10) the value for $1 + 2\kappa_1$ is found. The value found for $1 + 2\kappa_1$ is then used in any subsequent measurements on any test surface whose optical properties are unknown.

The apparatus in Fig. 2 was used to acquire interference pattern intensity data over an array of field points, which I then analyzed to calculate the phase-change-on-reflection values found in Table 1. The interference objective used was a 40× Mirau with a NA equal to 0.5. I empirically determined the value for $1 + 2\kappa_1$ to be 1.44, using the average handbook value for gold (0.617 rad) and dividing that value into the measured phase found for gold at the point of maximum fringe visibility in the interference pattern. The interference pattern was sampled at 28 frames per fringe. Note that the phase change on reflection for a material will generally increase as a function of oxide layer growth.

Summarizing, a microscope interferometer with an extended quasi-monochromatic illumination source can be used to extract the phase change on reflection for any given material by an analysis of the two-beam interference pattern sampled along the optical axis.

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