Spatial Coherence in Interferometry

Zygo’s new method to reduce intrinsic noise in interferometers

Michael Küchel

Abstract:
This paper describes a new light source, called the “Ring of Fire™ (RoF), with a specific spatial coherence function, which greatly reduces the intrinsic instrument noise. The evaluation of a series of very simple measurements taken with a conventional point light source and the RoF show clearly the very large gain in S/N ratio as a function of spatial frequencies obtained with this new light source.

Introduction
The introduction of the Laser as the standard light source for interferometers solved immediately the problems associated with the trade-off of low intensity and low temporal and spatial coherence of spectral lamps, and made it possible for George Hunter of Zygo to invent the GH interferometer, “that revolutionized the optics industry” [1]. That important invention facilitated the production of high accuracy optical parts worldwide in the past three decades.

Among the challenges of today in designing commercial interferometers is to reduce the intrinsic measurement noise in the complete band of spatial frequencies which the instrument is able to measure. Here the large coherence of the laser turns out to be a problem, “collecting” artifacts associated with the optical surfaces used in the system. The “common path” principle of Fizeau interferometry can solve that problem only partially; except for the very low end of spatial frequencies, the imperfections of the interferometer optics “print through” into the test result.

As a consequence, in recent years alternative light sources, mainly with lower temporal coherence, have received more attention [2] and scanning white light interferometers have been introduced for microscopic applications [3]. Low temporal coherence not easily combines with Fizeau interferometry [4], but the main problem is, that with a reduced temporal coherence only backward “scatter” is reduced, whereas forward “scatter” is still a problem (“scatter” is used here in a very general sense). The quantity which really causes the trouble is the high spatial coherence of laser sources, not their high temporal coherence.
This problem has been addressed by lowering the spatial coherence by replacing the “point-like” light source, which is not “resolved” by the aperture of the collimator, by a “disk-like” source. This can be implemented by having the laser illuminating a slightly defocused spot on a rotating ground glass. This Zygo invention [5] leads to sub-nm precision in laser-based interference microscopy. For Fizeau-type interferometers with unequal path lengths, there is a trade-off of the amount of coherence reduction and a reduction of the contrast of the interference fringes.

Studying the reasons for the reduction in contrast of a disk-like light source lead to a new invention [6], which is a light source with the shape of a ring concentric to the optical axis. This invention, called the “Ring of Fire™, RoF, is introduced in new Zygo products [7] and preserves the benefits of laser Fizeau interferometry and at the same time reduces artifacts and intrinsic noise by a very large amount. In contrast to all other known methods, this technique does not impose restrictions on to the cavity length, preserving the optimal visibility of the interference fringes. It will be shown that the intrinsic instrument noise in the measurements is now extremely small, 0.05nm r.m.s.. But even more important is the large gain in S/N ratio for the complete band of spatial frequencies the instrument is intended to measure. We will now explain how this works and give experimental evidence.

The optical arrangement of a Fizeau interferometer and the common path principle
In order to describe how the spatial coherence of the light source illuminating a Fizeau interferometer influences the optical (coherent) noise-contributions to the image forming waves, we will first establish the basic optical arrangements of a Fizeau-interferometer. This is done with the help of Figure 1.

Figure 1: Schematic arrangement of a Fizeau interferometer
A simple model of an interferometer structures it into three parts:

1. the illumination part, associated with the “illumination train” or imaging of the light source,
2. the Fizeau cavity
3. the imaging part, associated with the “imaging train” or imaging the test surface onto the detector.

The illumination part of the interferometer consists of the point light-source L, (which is in the Figure 1 the end of a monomode-fiber illuminated by a laser with high temporal coherence), the beam splitter and the collimator. The collimator images the light source to infinity, so that all rays falling onto the Fizeau cavity are parallel to the optical axis and perpendicular to the test surface and the reference surface.

The Fizeau cavity is built by the test surface and the reference surface and the index of refraction of the gas filling that volume.

The “imaging part” consists of the collimator lens and the ocular lens with beam splitter in between and the CCD detector behind the ocular lens. The imaging aperture is located in the common focal plane of the collimator lens and the ocular lens. This arrangement is a Kepler telescope, which is an “afocal system” with the special feature that it not only images an object at infinity into an image at infinity (see the imaging of the light source), but in addition a real object placed in front of the first lens system is imaged into a real image behind the second lens system. This is illustrated in figure 1 by arbitrarily choosing point A with its image point A’ on the detector.

Generally the optical configuration can be seen as a microscope with spatially coherent, i.e. “point” illumination (L), where the object to be investigated (in the microscope it is transparent) is replaced by the reflective part under test, which is the test surface. In addition, the reference surface acts like another (reflective) object, which is now – more or less – out of focus. The purpose of the microscope is to image the object onto the camera. The purpose of the interferometer is to make the topography of the surface under test measurable. Nevertheless, the ability of an interferometer to image the test surface onto the detector without “smoothing” the fine details on the surface is a very important quality criterion for an instrument, because it enables to measure mid-spatial frequency and high spatial frequency surface features. It can be expressed with the “Instrument Transfer
Function”, ITF, which quantifies the ability to transfer the amplitudes of the surface into the measured phase-map as a function of spatial frequencies. The reference surface is also imaged onto the detector, but this image is out of focus, which is no drawback, since the topography of the reference surface is not the measurand.

In a simplified description of the Fizeau interferometer the measured phase-difference is associated completely with the optical path difference $2d(x,y)$ introduced into the system by the Fizeau-cavity, i.e. it is described as:

$$\varphi(x, y) = k \cdot 2d(x, y) \quad \text{with} \quad k = \frac{2\pi}{\lambda}$$

where $d(x,y)$ is the geometrical distance between the reference surface and the test surface as a function of the test surface lateral coordinates $x,y$, $k$ is the wave number and $\lambda$ is the wavelength used for the test. Here the air filling of the cavity is assumed to have a homogenous index of refraction and is neglected.

Often the explanation is given that the interference is formed in the cavity and that the interference fringes are then “imaged” onto the detector. This description is valid only approximately; in a rigorous discussion, the interference takes place at the detector which measures the square of the sum of the complex amplitudes of test wave and reference wave. This amplitude sum depends on the optical path differences of the two waves which must be measured from the point light source via the test surface to the detector for the test wave and from the point light source via the reference surface to the detector for the reference wave. Only in the case that the test surface and the reference surface are “identical” in their topography, that they are not tilted to each other and that the cavity length is very small (these 3 conditions can never be fulfilled perfectly), the optical paths of test and reference beam are exactly identical in the illumination part and in the imaging part of the interferometer. In this ideal case they would cancel completely due to this “common path principle” of the Fizeau interferometer, and a very clean interferogram would result.

One can imagine that in practice a great number of surfaces is needed to build a well designed Fizeau interferometer, i.e. with low wavefront aberrations and good imaging features. There easily could be 15 surfaces in the illumination part and 20 surfaces in the imaging part. This “practical side” of designing an interferometer was ignored in the more theoretical discussions so far. Now, it is obvious that the complex amplitudes carrying in their phases the information about the topography of the test surface and the reference surface
will be heavily altered when transmitted by the many surfaces in their way to the CCD detector. The coherent “noise” that is added in the imaging part to the illuminating wavefronts, which are already not ideally “plane” (in the sense that the phase will contain already artifacts from the so far transmitted surfaces), will in sum be larger than the signal, leading to a S/N ratio < 1 for both waves individually. The common path principle now “cleans” the signal (which is the difference of phases) by removing those noise contributions which are identical in both complex amplitudes. Practically this condition is met best for the low-spatial frequency part in the added complex amplitudes, but not very well for the mid-spatial frequency part and never for the high-spatial frequency part of the spectrum. We mentioned already two reasons why the common path principle is violated: a deviation of the surface figure of the test surface from the reference surface (and this is always given) and a tilt between the surfaces\(^1\). We expect therefore to get an increasing contribution of “intrinsic noise” by increasing tilt. This was the idea behind the measurement series presented later in this paper, to quantitatively measure the effect of the RoF on the suppression of intrinsic noise by measurements with increasing tilt and subtraction of the cavity!

\(^1\) That is the reason why the customers are told to “null” their cavity!
How the RoF works

The idea for the RoF was born when the reason for the lost in contrast in a Fizeau-cavity with a temporal coherent, but spatially extended light source was studied. The following Figures 2-4 explain the problem.

Figure 2: The optical path difference of point A, \(\text{OPD}_A\), introduced in the Fizeau cavity is a function of the off-axis position \(y\) of the illuminating point source. The ray being reflected at the reference surface \(R\) and interfering with the ray from object point A on the test surface \(T\) hits the reference surface at point \(Q_A\).

Figure 3: Geometry to calculate \(\text{OPD}=\text{OPD}(\alpha)\).

Figure 4: Gauß’s definition of focal length.

It was well known, that a spatially extended light source, a small disk with radius \(y_{\text{max}}\) centered at the optical axis and replacing the point light source, suppresses the intrinsic noise very nicely [5]. Such a light source is known to be spatially incoherent, and it has the drawback that at the same time when the coherent noise and artifacts are suppressed the visibility of the interference fringes is reduced. This reduction in visibility increases with increasing length \(d\) of the Fizeau cavity. The effect can generally be explained by the application of the van Cittert Zernike Theorem [9], but to make it very easy, we compute the optical path difference \(\text{OPD}_A\) in the cavity for an arbitrary object point A on the test surface as a function of the off-axis position \(y\) of a point-source in the focal plane of the collimator. The beam splitter shown in Figure 1 is omitted here for simplicity. Figures 2, 3 and 4 show the
details: the spherical wave diverging from point L is collimated by the collimator lens and a plane wavefront forming an angle $\alpha$ with the optical axis illuminates the plane parallel cavity of surfaces T and R. After the round trip of the ray illuminating point A on the test surface, it combines with another ray from the light source in point $Q_A$ and both rays travel to the image $A'$ on the CCD detector. All points on the plane wavefront in $Q_A$ have the same optical path from source point $y$, and it is obvious that the test ray and the reference ray from $Q_A$ to $A'$ coincide and therefore have identical optical paths; the only path difference is introduced in the optical cavity, and it is calculated to be:

$$OPD_A = 2d \cos(\alpha)$$

(2)

where $\cos(\alpha)$ can be calculated from:

$$y = f \cdot \tan \alpha$$

(3)

$$\cos \alpha = \frac{1}{\sqrt{1 + \tan^2 \alpha}} = \frac{1}{\sqrt{1 + \frac{y^2}{f^2}}} \approx 1 - \frac{1}{2} \frac{y^2}{f^2}$$

(4)

$$OPD_A(y) = 2d \cdot \left(1 - \frac{y^2}{2f^2}\right)$$

(5)

Therefore one can calculate the change $\Delta OPD_A$ in the $OPD_A$ when the light source is moved laterally from position $y_0=0$ on the optical axis to a new off-axis position $y$:

$$\Delta OPD_A(y) = OPD_A(y) - OPD_A(0) = 2d \cdot \left(1 - \frac{y^2}{2f^2}\right) - 2d = -d \frac{y^2}{f^2}$$

(6)

Equation (6) shows that a lost in contrast must be found when a complete disk with the maximal radius $y=y_{\text{max}}$ is filled with light. As the phase-difference $\phi$ and the OPD are related to each other through $\phi = k \cdot OPD$, the intensity of an interferogram for the arbitrary point A on the test surface, when the point light source takes on off-axis position $y$, is given by:

$$I_A(y) = I_T + I_R + 2 \sqrt{I_T I_R} \cos(\phi_A - \Delta \phi_A) = I_T + I_R + 2 \sqrt{I_T I_R} \cos \left(\phi_A - \frac{kd}{f^2} y^2\right)$$

(7)

where $I_T + I_R$ is the incoherent sum of the intensities reflected at the test and reference plate and the term $2 \sqrt{I_T I_R}$ is the modulation. If now the complete disk with radius $y_{\text{max}}$ is homogeneously filled with light, the intensity is achieved by an integration over that disk [10, 11]:
The reduction in fringe visibility is therefore a quadratic function of the source disk radius \(y_{\text{max}}\), or more meaningful, of the illumination aperture \(y_{\text{max}}/f\), and in addition linearly dependent on the cavity length \(d\). The first zero of the sinc-function is obtained when the argument is \(\pi\), therefore the visibility vanishes for a cavity length \(d_{\text{zero}}\) of:

\[
\frac{2\pi \cdot d_{\text{zero}} \cdot y_{\text{max}}^2}{4\pi \cdot \lambda \cdot f^2} = \pi \quad \Rightarrow \quad d_{\text{zero}} = \frac{2\pi \cdot \lambda \cdot f^2}{y_{\text{max}}^2}
\]

(9)

For a source disc radius \(y_{\text{max}} = 1\)mm and a collimator focal length of 500mm, as well as \(\lambda = 633\)nm, the Fizeau fringes vanish totally for a cavity length of 994mm; reasonable visibility of >0.64 is obtained up to a cavity length of 500mm. This restricts the general use of the concept of an extended source, especially for a commercial interferometer, where different customers have different requirements.

But looking at equation (6) again, one can state some remarkable feature:

1. One can (dynamically) move the point light source laterally in order to phase-shift the cavity.
2. If the cavity is mechanically phase-shifted in addition, one can dynamically move the light source without losing contrast during phase shift. For instance, when the point is quickly moved within the integration time for one camera frame on a spiral with increasing radius and the change in OPD associated with the increased radius is compensated by an opposite change of the cavity length \(d\) by a PZT (piezo-transducer) or alternatively by wavelength tuning, one can create a disc source without the loss in contrast.
3. Another remarkable observation in equation (14) is that the quantity \(y\) always is squared, i.e. \(\text{OPD}_A\) or \(\Delta\text{OPD}_A\) remain the same when the point light source is in the opposite location of the optical axis. It is also obvious, that the complete problem must be symmetrical with respect to the optical axis. In other words, when the light source consist of a ring of points, with the radius \(y\), then the \(\text{OPD}_A\) in the cavity for point A is constant for all points on that ring. Therefore the contrast of the fringes must be preserved!

Possibility 2. must work, but is technically more complex. Therefore the question is, whether the ring-source, possibility 3, will adequately suppress intrinsic noise and artifacts. To answer
that question, we investigate Figure 5, which adds to Figure 2 the case of the opposite position of the point source.

The remarkable fact is, that the chief-rays imaging the point A into point A' take very different paths through the optical system, when the source point L is moved on a circle around the optical axis. The same is true for the chief ray reflected on the reference surface: the two points P_A and Q_A shown in Figure 5 corresponds to a circle on the reference surface, with the radius

\[ \rho_R = d \tan \alpha = \frac{d \cdot y}{f}. \]

From investigating figure 5, one may observe that similarly other circles with larger or smaller radii are covered by the other rays, when the illumination point spins around a circle with radius y. The only points which remains stationary all the time are the points A and A'!

Now it should be obvious that such a ring source must suppress coherent artifacts very nicely: the background noise is generated by the roughness of the optical parts which are transmitted by the rays. Now the surface profiles are convolved with circles, which are the projections of the RoF onto the surfaces. This is an integration process which is known to
smooth noise, and could be calculated with the help of the auto-correlation functions of the surfaces.

In a different view, one could think how the different spatial components of the test surface are distributed in the imaging aperture, the low spatial frequencies near the optical axis and the higher spatial frequencies more to the edge of the aperture. The aperture together with the transfer function from the camera filters these components. The same is true for the additional complex amplitudes (noise) added to the signal from the individual surfaces in the system: the ITF for the complex amplitude will already reject some noise components, but now the complex amplitudes from the individual surfaces are first convolved with the RoF before the result is multiplied with the ITF. After the convolution the signals have already become very small.

**Experimental results**

Zygo has developed a Fizeau interferometer including a RoF, called the VeriFire AT™. The specification of this instrument can be found on Zygo’s web site [12]. The ring is achieved by a special hologram. This instrument includes the possibility to change between point source and the RoF, thus making direct comparisons very convenient. The following two Figures 6a and 6b show an example of two phase-maps; here the typical difference is clearly visible in the much better suppression of artifacts and noise, like the “bulls eye pattern” associated with dust particles or surface defects.

![Figure 6a: phase map with point source](image1.png) ![Figure 6b: phase map with RoF](image2.png)
In order to quantify the difference, we took a measurement series with a very clean VeriFire AT 1000 system, having a 1k x 1k CCD detector and a beam diameter of 4 inch.

Two measurement series where taken, one with the point source and a second one using the RoF. The two measurement series consisted of 2 x 50 individual phase-maps, each phase-map being an average of 16 phase-measurements, which were derived with the Zygo 13 bucket algorithm. The measurement series were taken in one of Zygo’s laboratories where the temperature of 23.2 degree Celsius was stable to within 0.1 degree Celsius and the total measurement time was about 2 ½ h. A plane cavity was set-up with a very small gap of about 2mm between a transmission flat, TF, and another flat surface acting as the test surface. No special care was taken to use surfaces with small figure errors because the goal of the measurement series was to measure the intrinsic noise of the system with point source and with RoF, rather than to measure the phase-map of the cavity. This was achieved by the following special measurement sequences, as shown in Table 1, which was performed twice, for the point and the RoF. The only other change in the series was the tilt of the test surface, which was changed in many steps between +5 fringes to –5 fringes of tilt; the reference surface was not touched during the complete series of measurements.

<table>
<thead>
<tr>
<th>tilt of test surface</th>
<th>1 fringe: smaller cavity on top; repeat 5 times</th>
<th>2 fringes: smaller cavity on top</th>
<th>3 fringes: smaller cavity on top</th>
<th>4 fringes: smaller cavity on top</th>
<th>5 fringes: smaller cavity on top</th>
</tr>
</thead>
<tbody>
<tr>
<td>tilt of test surface</td>
<td>(1) 1 fringe: smaller cavity on top; repeat 5 times</td>
<td>(3) 2 fringes: smaller cavity on top</td>
<td>(5) 3 fringes: smaller cavity on top</td>
<td>(7) 4 fringes: smaller cavity on top</td>
<td>(9) 5 fringes: smaller cavity on top</td>
</tr>
<tr>
<td>diff. (A)</td>
<td>A1={(2)–(1)}/2</td>
<td>A2={(4)–(3)}/2</td>
<td>A3={(6)–(5)}/2</td>
<td>A4={(8)–(7)}/2</td>
<td>A5={(10)–(9)}/2</td>
</tr>
<tr>
<td>sum (B)</td>
<td>B1={(2)+(1)}/2</td>
<td>B2={(4)+(3)}/2</td>
<td>B3={(6)+(5)}/2</td>
<td>B4={(8)+(7)}/2</td>
<td>B5={(10)+(9)}/2</td>
</tr>
</tbody>
</table>

Table 1: Measurements taken and basic evaluations: take the differences and sums of measurements taken with the same amount of tilt with opposite sign!

The idea behind these measurements and the evaluation is the following: when the common path principle cancels the intrinsic noise of the system, this noise must show-up when the common paths principle is violated. In two measurements with opposite tilts in a very small plane cavity, the OPD in the cavity itself has not changed other than for the tilt; therefore if two such measurements are subtracted from each other, the OPD of the cavity cancels. The remainder is the intrinsic noise added by the system, which is a function of the amount of differential tilt. Note that the intrinsic noise produced by the reference surface cancels in this procedure in addition (because the reference surface was not touched), so we only see the
intrinsic noise produced from the test wavefront. We performed two analysis on these phase-map differences (A) as shown in the table: we computed the r.m.s. values as a function of the differential tilt for the point source and for the RoF, see Figures 9a, 9b and we computed the gain in dB for the measurement of surface height features\(^2\) with the RoF over those obtainable with the point-source as a function of spatial frequencies, see Figure 10.

Both evaluations are especially suited for the characterization of the quality of a large aperture interferometer, as they provide insight in which range of spatial frequencies the measurement are reliable and from which frequency on the noise starts to be the limiting factor for the usefulness of the measurements. Also, understanding these results will help a customer to plan special measurement procedures for his most demanding applications where spatial averaging is incorporated to further improve the S/N ratio of his measurements. But the results clearly show the tremendous gain by the RoF technology.

In the evaluations (B) of the same measurement data the OPD of the cavity is computed, where now both the intrinsic noise from the test amplitude as well as from the reference amplitude is contained in addition to the phase-map of the cavity. These measurements are all averaged to get a result with reduced noise-content, because by the 10 different tilt positions the artifacts change and therefore average to a smaller value. This result is then used to give a reference for the intrinsic noise found before. Building again an amplitude ratio allows the computation of a S/N ratio as a function of spatial frequencies, i.e.

\[
S/N = 10 \cdot \log_{10} \left( \frac{\sqrt{PSD_P(f)}}{\sqrt{PSD_R(f)}} \right)
\]

is computed, where now the average value of the evaluation (B) is associated with the "signal" and the result from evaluations (A) serve as "noise", expressed now in the subscript for PSD. This is shown in Figure 11 only for the case of the RoF.

\(^2\) To be specific: for Fig. 10 we compute: \(PSD = \frac{1}{s} \left( FFT(A) \cdot FFT^*(A) \right) \) where s is a normalization constant, FFT is the Fast Fourier Transformation and the star defines the complex conjugate. We do this for the point source, subscript P and for the RoF, subscript R. As surface height profiles are amplitudes, we now compute the gain in suppression of intrinsic noise as a ratio, expressed in dB, as: \(gain(f) = 10 \cdot \log_{10} \left( \frac{\sqrt{PSD_P(f)}}{\sqrt{PSD_R(f)}} \right) \), where \(f\) is the spatial frequency.
Figure 7a: Example for intrinsic noise phase-map: difference for ± 5fringes, point source. Different types of artifacts are visible: on a background of “random noise” from the roughness of the surfaces, we see “bulls eye patterns” from particles, higher order reflections creating a vertical high frequency interference pattern as well as “bands” were the contrast of this high frequency pattern is zero. In the very center we see in addition the tiny “ghosts” from the lens centers.
Figure 7b: as Figure 7a, but RoF light source. The grey scale is kept the same as in figure 7a. Only small disturbances are left, especially some remaining “ghost” from the lens centers. In addition it can be seen, that some very low order aberrations are present: the grey-tones are not completely homogeneous across the field. This observation was confirmed by Figures 9a and 9b.
Figure 8a: $\sqrt{\text{PSD}}$-function of 7a.

Figure 8b: $\sqrt{\text{PSD}}$-function of 7b.
Figure 9a: RMS values of intrinsic noise as a function of the difference angle. The extrapolated value for zero difference angle reflects the fact that the surfaces in the cavity never match perfectly, so the common path principle never is fulfilled completely! Also, there are other noise contributors influencing the reproducibility.

Blue: RoF, green, point source. Note how small the values are! This shows the extremely high standard of this instrument!

Figure 9b: as Fig. 9a, but now after subtraction of the first 9 Zernike terms Note that a part of the r.m.s. error in Fig. 9a was caused by these low order aberrations. For the RoF the r.m.s. after subtraction of the 9 Zernike terms is independent of the tilt. This enables to achieve low uncertainty measurements also on parts with a larger figure errors.
Figure 10: Gain in S/N ratio in the measured phase maps for RoF over point as a function of spatial frequencies for an angle between test and reference beam of 31.2μrad. The measurements were performed 5 times. Very significant gain for the S/N for spatial wavelength between 20mm and 0.5mm.

Figure 11: S/N ratio measuring the PSD function of the cavity in the case of the RoF as light source. Note: a very large S/N ratio > 15 dB is obtained for the important band of spatial frequencies up to 1/mm (10 pixels/period). At the low end, the S/N ratio is limited by mechanical vibrations introducing artifacts for the phase-measurement, for highest frequencies the detector limits the S/N ratio. Near Nyquist the ratio is <1, i.e. noise is dominating. Note: referenced to a very good cavity with a deviation of only 1.7nm r.m.s..
Acknowledgement

I want to thank my colleagues at Zygo for developing and realizing the system with the RoF, and my colleagues Maria Robinson and Michael Schmidt for setting up the instrument and performing the measurements, and my colleague Thomas Dresel for critical reading this manuscript.

Literature

   Küchel et al: Reducing Coherent Artifacts in an Interferometer, international patent application WO 02/090882 A1, (Publication Date 14.11.2002),
[7] VeriFire AT™
[12] www.zygo.com