R&D Technical Bulletin

P. de Groot **9/23/93**

What is Frequency Domain Analysis?

Abstract: The Zygo *NewView 200* is a scanning white-light interferometer that uses frequency domain analysis (FDA) to generate quantitative 3D images of surfaces. FDA is a mathematical method for processing complex interferograms in terms of phases and spatial frequencies. This memo is a tutorial description of FDA to facilitate understanding the fundamental ideas behind the technique.

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Introduction

What is FDA? Frequency-domain analysis (FDA) is a way of processing interferograms to obtain surface profiles. When we say that we are analyzing data in the frequency domain, it means that we are thinking about the different phases and optical frequencies that contribute to a fringe pattern created by an interferometer.

For example, in the old days, interferograms of optical surfaces were often painstakingly analyzed by hand, using a photograph of the fringes. It is now much more common to perform some form of phase shifting interferometry (PSI), which allows us to *transform* the interference pattern electronically into a matrix of phase values. These phase values can then be directly related to relative height values, provided that the wavelength or optical frequency of the source light is known. The transformation from fringes to phases is accomplished with an algorithm such as the familiar five-bucket method.

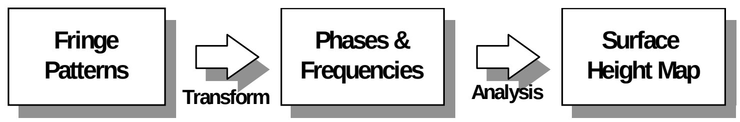

Figure 1: This is FDA. Data are transformed to the frequency domain, where it is analyzed to calculate surface height.

The operating principle of the Zygo *NewView 200*[*] is a natural and logical extension of PSI methods that have proven so accurate and reliable in the past. Just as is done with PSI, we mechanically scan the objective lens of an interferometric microscope to generate a sequence of interferograms, which are transformed to get phase information. However, the interferograms generated by the *NewView* are far more complex, because of its white-light source. This complexity is to our advantage, since there is far more information about surface structure in the white-light interferogram.

In the *NewView* we use Fourier Analysis to extract a range of phases for each color or wavelength in the spectrum of the source. The source spectrum together with the corresponding phases is said to be a *frequency domain* representation of an interferogram. One way to think of this is to imagine applying the five-bucket algorithm for every one of the wavelengths in the source spectrum. The resulting phase measurements would be different for each wavelength. The particular combination of phases found by FDA uniquely defines the surface height map. This is what we call analysis in the frequency domain.

[*] US Patent No. 5,398,113. Additional US and Foreign Patents Pending.

The following two sections describe the conceptual and mathematical foundations of FDA, starting with the simple example of a single-wavelength system and working up to *NewView* technology.

PSI: FDA with a single wavelength

Let's look in detail at how FDA works in traditional laser interferometers. In Figure 2 we see a sketch of a Michelson interferometer, composed of a beam splitter, a reference mirror and an object mirror. The round-trip optical path difference (OPD) between the two mirrors is L. Generally, the distance $L/2$ to the object is what we are interested in measuring.

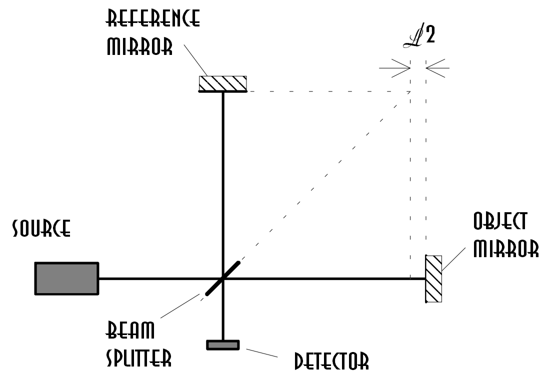

Figure 2: Michelson interferometer.

When the object mirror is scanned backwards so as to increase the OPD L, the intensity measured by the detector varies sinusoidally. The relationship between intensity and object distance is illustrated in Figure 3. For a well-balanced interferometer with a single-frequency source, the normalized intensity varies as

$$I = \tfrac{1}{2} \cdot \left(1 + \cos(\varphi)\right),$$

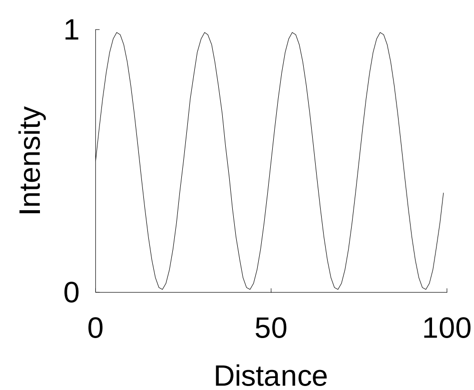

Figure 3: Variation of measured intensity with object distance in an interferometer.

where φ is the interferometric phase. The phase is related to the OPD L, the source wavelength λ, and a constant phase offset φ_0 characteristic of the interferometer and the material properties of the mirrors. In writing down this relationship, it is convenient to introduce a quantity $k = 2\pi/\lambda$, known as the wavenumber or spatial frequency of the source light. Thus

$$\varphi = L \cdot k + \varphi_0 \ .$$

The interferometric phase is a familiar linear relationship of the form $a \cdot x + b$ encountered in algebra. Such a linear relationship can be easily represented in graphical form. Let the spatial frequency k be the abscissa of a graph, and the phase φ be the ordinate. The interference pattern can be drawn as a line having a slope L and an intercept φ_0. Using this *spatial-frequency domain* representation, we can easily find the phase for any given spatial frequency k, or conversely, if we know the phase φ, the spatial frequency k and the offset φ_0, we can figure out the slope L of the line.

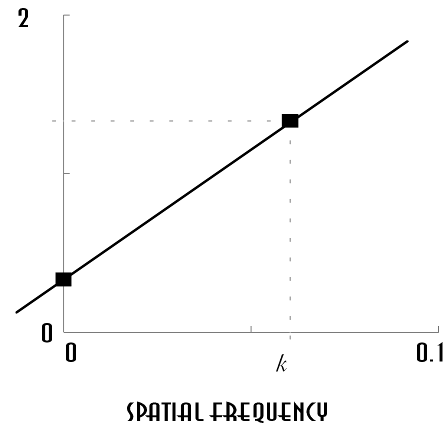

Figure 4: Graphical representation of single-wavelength interference in the spatial frequency domain.

In PSI, we generally know the wavelength λ and therefore the spatial frequency k of the source. Also, we can determine or arbitrarily fix a value for the offset φ_0. The remaining problem is how to measure the phase φ. This is done by taking data at a few sample points on the curve shown in Figure 3, and *transforming* the data into the frequency domain by means of a simple algorithm that can be implemented on a computer, such as the five- or eleven-bucket phase calculations. The OPD is calculated from the slope of the graph, using the formula

$$L = \frac{\varphi - \varphi_0}{k}.$$

The essential idea behind FDA is that we are looking at the interference phase, not the original intensity data measured by the detector (Figure 1.) To get from the intensity data to the frequency domain, we transform the data using a mathematical calculation. Distance is calculated by determining the slope of the line showing the rate of change of interferometric phase with spatial frequency.

We will now look at how FDA is applied to more complex forms of interferometry, and in particular, to the *NewView 200*.

The *NewView 200*: FDA with White Light

Although traditional, single-wavelength PSI is a very accurate way of measuring surface heights, it has important limitations. Phase calculations in PSI can only be performed unambiguously within a range of $\pm\pi$, corresponding to a surface height range of $\pm\lambda/4$. Outside this range, the measurement is *modulo* 2π and there may be several possible slopes L for the same phase measurement. This is illustrated by the frequency-domain graph in Figure 5.

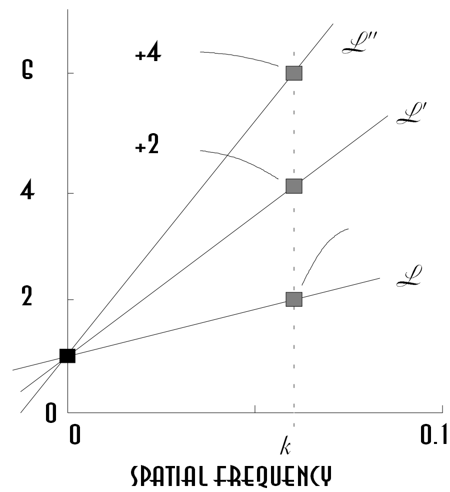

Figure 5: The 2π ambiguity problem--which slope is it?

One way to look at this problem is to say that there just aren't enough points on the graph to uniquely define the slope. We need at least one more point, or better yet a range of points, calculated for different spatial frequencies. Then there would be only one possible line in the frequency domain graph.

Any source with a large, continuous range of spatial frequencies can be loosely described as a white-light source. Such a source is also said to have a *broadband spectrum*, composed of a continuous band of colors or wavelengths. White light interferograms tend to be more complex in appearance than the single-wavelength example shown in Figure 3, because a whole range of sinusoidal patterns are superimposed on each other.

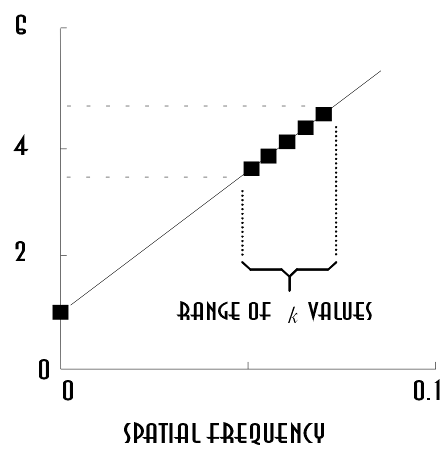

Figure 6: The *NewView 200* solves the ambiguity problem with white light, which provides phase information for a range of spatial frequencies k.

Despite the complexity of white-light interferograms, it is still possible to extract phases for the individual spatial frequencies contributing to the interference effect. This is done by means of a *Fourier Transform*, or more specifically, by the computationally efficient *Fast Fourier Transform*. The phase information in the Fourier-transformed data can be plotted exactly as shown in Figure 6.

Once the data have been transformed into the frequency domain, we can measure the object distance by determining the slope L of the line drawn through the data points in the frequency-domain graph. The *NewView 200* uses a simple linear least-squares fit to several phase values having high signal-to-noise ratio. The final distance calculation can be done in either one of two ways, depending on whether or not we include a phase offset φ_0 in the calculation. The *normal mode* uses only the least-squares slope result and is recommended for rough surfaces, while the *high resolution mode* makes use of all of the available information for maximum precision.

How the data are collected

The *NewView 200* is a scanning white-light interferometer. The instrument includes appropriate optics for imaging an object surface and a reference surface together onto a solid-state imaging array, resulting in an interference intensity pattern that can be read electronically into a digital computer. Interferograms for each of the pixels or image points in the field of view are generated simultaneously by scanning the object in a direction approximately perpendicular to the surface illuminated by the interferometer, while recording detector data in digital memory. The data acquired in this way consists of an array of interferograms, one for each pixel, representing the variation in intensity as a function of scan position. The interferograms stored in the computer are individually processed by FDA, and the final step is the creation of a complete three-dimensional image constructed from the height data and corresponding image plane coordinates.

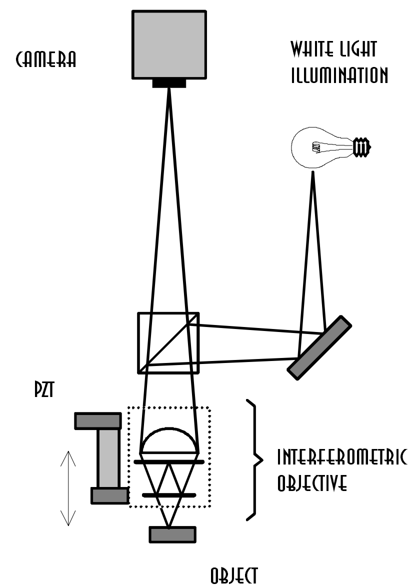

Figure 7: Optical system for scanning white light interferometry.

The *NewView* FDA software is based on a computationally efficient implementation of the Fast Fourier Transform and is capable of very high speed. For large surface features, this high speed is augmented by means of a sophisticated data discriminator, which rejects those portions of the interferograms that have very low fringe contrast.* The data discriminator greatly reduces the amount of processing required to render an accurate three-dimensional image of the surface. This feature is critical to the successful implementation of FDA on the *NewView*.

* US Patent No. 5,402,234. Additional US and Foreign Patents Pending.

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Advantages of FDA for surface structure analysis

The Zygo *NewView 200* uses frequency domain analysis because it...

- makes efficient use of all available interference data;
- is relatively insensitive to noise such as spikes, gaps or variations in DC bias;
- easily accommodates variations in source characteristics such as mean wavelength;
- is insensitive to changes in surface characteristics such as color and brightness;
- is accurate and computationally efficient.

These advantages are obtained by working with interferometric phases in the spatial frequency domain, according to the same principles that have made phase-shifting interferometry so successful. Thus we have preserved the accuracy, flexibility and precision of PSI while greatly extending the operational range and functionality of the instrument.

For Further Information…

1. P. de Groot and L. Deck, "Three-dimensional imaging by sub-Nyquist sampling of white-light interferograms," Opt. Lett. 18(17), 1462-1464 (1993).

2. L. Deck and P. de Groot, "A new high-speed non-contact profiler based on scanning white light interferometry," Proc. ASPE 424-426 (1993).

3. P. de Groot and L. Deck, "Surface profiling by frequency-domain analysis of white-light interferograms" Proc. SPIE 2248, Optical measurements and sensors for the process industries, 101-104 (1994).

4. P. de Groot and Leslie Deck, "Surface profiling by analysis of white-light interferograms in the spatial frequency domain," J. Mod. Opt. 42(2), 389-401 (1995).

5. L. Deck and P. de Groot, "High-speed non-contact profiler based on scanning white light interferometry," Appl. Opt. 33(31), 7334-7338 (1995).

6. S. Chakmakjian, J. Biegen and P. de Groot, " Simultaneous focus and coherence scanning in interference microscopy " Proceedings IWI, (Riken, Japan, 1996).

7. J. Roth and P. de Groot, " Wide-field scanning white light interferometry of rough surfaces," Proc. ASPE Spring Topical Meeting (1997, *to appear*).